Quantum clocks and the foundations of relativity

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ABSTRACT

The conceptual foundations of the special and general theories of relativity differ greatly from those of quantum mechanics. Yet in all cases investigated so far, quantum mechanics seems to be consistent with the principles of relativity theory, when interpreted carefully. In this paper I report on a new investigation of this consistency using a model of a quantum clock to measure time intervals – a topic central to all metric theories of gravitation, and to cosmology. Results are presented for two important scenarios related to the foundations of relativity theory: the speed of light as a limiting velocity and the weak equivalence principle (WEP). These topics are investigated in the light of claims of superluminal propagation in quantum tunnelling and possible violations of WEP. Special attention is given to the role of highly non-classical states. I find that by using a definition of time intervals based on a precise model of a quantum clock, ambiguities are avoided and, at least in the scenarios investigated, there is consistency with the theory of relativity, albeit with some subtleties.

Keywords: quantum tunnelling, superluminal propagation, principle of equivalence, tunnelling time, theory of relativity

1. BACKGROUND

Twenty-first century theoretical physics rests on the twin pillars of quantum mechanics and the theory of relativity. It is well known that these subjects are uneasy bed-fellows. Already in the 1930's the proliferation of divergencies began to plague quantum electrodynamics, and although they were eventually tamed by systematic renormalization techniques, the procedure has always had an air of fudge[1]. The further application of quantum field theory to gravitation ran into intractable problems of non-renormalizability, so that it is only with the development of esoteric formalisms such as string theory, M theory or loop quantum gravity that the fundamental tension – one might even say incompatibility - between quantum mechanics and metric theories of force promises to be finally removed[2].

This tortured history has tended to conceal a more basic clash between the conceptual foundations of quantum mechanics and relativity theory. In the early 1980's I had an interesting conversation with John Wheeler while walking across the Yorkshire moors. We mused about how the formulation of the theory of relativity might have been different had quantum mechanics come before it. Indeed, what if quantum mechanics had preceded classical mechanics and the Galilean principle of relativity? The problem is easily stated. The theory of relativity in both its special and general formulations depends crucially upon the inter-comparison of distances and time intervals, using measuring rods and clocks. These concepts are well defined in classical physics, but in quantum mechanics deep problems emerge. Particles do not follow precise world lines. Attempts to locate particles in space using wave packets are complicated by the fact that those packets spread. Worse problems afflict attempts to measure intervals of time. Time is not an operator in quantum mechanics; rather it is a parameter. How to measure it at all is a non-trivial question[3]. Quantum clocks are subject to inherent uncertainty, as has been clear since the pioneering work of Wigner[4].

These problems may be traced back to the fact that quantum mechanics is a non-local theory, while the theory of relativity – indeed, any metric theories of gravitation – are based on local measurements. The problem becomes especially acute in general relativity, where the principle of equivalence is tied to local freely-falling frames. Consider, for example, the experiences of a uniformly accelerating observer[5]. In unbounded Minkowski space in the standard vacuum state of the electromagnetic field, an accelerated observer (or particle detector) with proper acceleration g will perceive a bath of thermal radiation with a temperature

$$T = g\hbar/2\pi kc. \qquad (1)$$

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By contrast, consider a localised observer at rest on the surface of a spherically symmetric massive shell of radius r_s with local gravitational acceleration g. Suppose the quantum state is the Minkowski vacuum for $r < r_s$ and the so-called Boulware vacuum[6] (the natural generalization of the Minkowski vacuum to the spacetime region outside a static mass) for $r > r_s$. Then it is readily shown[6] that the observer will detect *no* radiation. Thus the principle of equivalence fails in this simple case, the reason being that the Minkowski and Boulware vacuum states are defined on the entire spacetime manifold, whereas the observations (e.g. particle detection events) are made along a locally-defined world line. There is generally no "natural" procedure for adapting the local to the global[7]. Thus the *same* quantum state will be experienced *differently* by observers (or particle detectors) that move differently.

The above difficulties arise when quantum mechanics is applied to problems of motion in a prescribed background spacetime. They are considerably worse when the spacetime itself is quantized. Although the serious disruption of spacetime (e.g. the appearance of spacetime foam) is likely to be restricted to Planck dimensions, it is possible that quantum fluctuations of spacetime can produce measurable effects in astronomy, cosmology or cosmic ray physics[8-10]. It has also been suggested that they may lead to modifications of the fundamental relation between the de Broglie wavelength and momentum[8], or the energy-momentum uncertainty relations[8], especially at high energies. In any case, the existence of quantum spacetime fluctuations places fundamental limits on the application of classical concepts of distance and time to the description of cosmology[8,10].

The further development of quantum gravity into string/M theory or loop quantum gravity suggests that time may be an emergent property of physics, and not a fundamental quantity[11]. Indeed, in quantum cosmology, time does not enter into the fundamental equations at all[12]. There is therefore a hierarchy of treatment. At the top is some yet-to-be achieved unified physics, in which quasi-classical spacetime might emerge at low (i.e. sub-Planck) energy from a fully quantum treatment. Then there is the semi-classical approach: quantum field theory applied to a prescribed classical background spacetime[6]. This theory has led, for example, to Hawking's prediction of black hole evaporation[13]. Then finally there is the lowest level of the hierarchy – ordinary quantum mechanics applied in a background gravitational field. Experimentally, the hierarchy is inverted. There are as yet no experiments or observations that reliably test the top level. There is some possibility that we may test the second level, for example, by observing black holes exploding in the final stages of the Hawking process. But the bottom level is accessible by a number of experimental procedures that have been developed in the last two decades, including discussions of antimatter in free fall[14], and matter wave interference in gravitational fields and opto-gravitational cavities[15-18]. Additional proposals remain a lively field of investigation[19]. There is thus strong motivation for a careful theoretical examination of this regime.

2. A MODEL QUANTUM CLOCK

The foundations of relativity depend on the consideration of sets of particles that move in a well-defined manner (e.g. constrained to form a rigid body), and between which one may measure distances and time intervals with arbitrary precision. Quantum mechanics compromises that conceptual scheme. However, it does possess a satisfactory classical limit when particles are described by sharply-peaked wave packets and use is made of Ehrenfest's theorem. But what about the vast majority of quantum states that are not contrived to approximate the classical world? Is it possible to begin with highly non-classical quantum states and still be consistent with the theory of relativity, both special and general? And if so, is it possible to *formulate* the theory of relativity (or any metric theory of motion) using quantum concepts as the starting point? The latter ambitious project is beyond the scope of this paper[20,21], but I shall present evidence for an affirmative answer to the former.

The basis for this discussion is to consider certain non-classical states, namely, energy eigenstates of unbound particles, moving in a prescribed background spacetime, and to discuss the measurement of time intervals using a specific model of a quantum clock. By making use of a quantum clock, time intervals are converted into the displacement of a position variable, which may then be measured by well-understood means. Several suggestions for quantum clocks have appeared in the literature[22]. Here I restrict attention to an early proposal by Salecker and Wigner[23], and later elaborated by Peres[24]. This model deals explicitly with stationary states, for which the time enters as a changing phase: e^{iEt}. The clock consists of a rotor that begins in an initial state with a well-defined pointer angle, and runs ("ticks around the clock face") only when the particle traverses the space between two points interest, say x_1, x_2. The clock's Hamiltonian is $P(x)\omega J$ where P is a projection operator for the position of the particle in the interval $x_1 < x < x_2$, J is the angular momentum of the clock rotor and ω the angular velocity. The quantum clock measures the change in phase of the particle's wave function in traversing the interval $[x_1, x_2]$, defining a duration known as the *phase time*. Because one deals with stationary states, the clock measures only time *differences* between two events, not the absolute time of either event. This will turn out to be crucial in what follows.

To illustrate the Peres clock, consider a free particle of mass m and energy E moving to the right in one space dimension, in a momentum eigenstate e^{ikx} described by the time-independent Schrödinger equation. We wish to calculate the time of flight between two points separated by a distance L, i.e. the time of passage of the particle on the interval $0 < x < L$. This will yield an expectation value of the particle's velocity v. Suppose one were to start by observing the particle at $x = 0$ and then observing it some time later at $x = L$. A precise initial position measurement would result in a collapse of the wave function to a position eigenstate $\delta(x)$. This would imply an arbitrarily large uncertainty in the momentum, which completely compromises our attempt to measure the velocity. But if we relinquish the attempt to measure an absolute time of passage and accept instead the time *difference*, then sensible results are obtained. The Peres clock calculation proceeds as follows (full details are given by Peres[24]). First we compute the phase difference $\theta(E)$ in the wave function between the two points; this is $\theta(E) = kL$ where $k \equiv (2mE/\hbar^2)^{½}$. Next replace E by $E + \varepsilon$ where ε is the coupling energy between the particle and the clock, treated in first order perturbation theory. Now expand $\theta(E + \varepsilon)$ to first order in ε. The coefficient is T, the required expectation value of the time for the particle to traverse the distance L. In the above case $T = mL/(2mE)^{½}$. Defining the classical velocity $v = (2mE)^{½}$, the expected time of flight is

$$T = L/v \tag{2}$$

which is the same as the classical result.

To show how the use of a well-defined quantum clock can bring some clarity and consistency to the confused interface between quantum mechanics and relativity, let us repeat the foregoing for a Dirac particle of rest mass m. As is well-known, the Dirac equation possesses a single-particle interpretation in which there exists a velocity operator that commutes with both the momentum p and the Hamiltonian for a free particle of energy E. The eigenvalues are $\pm\, c$, implying that a Dirac particle moves back and forth at the speed of light (the so-called zitterbewegung phenomenon). This may be understood as a manifestation of the arbitrarily large momentum uncertainty occasioned by making a precise position measurement at $x = 0$. (An infinite momentum corresponds to a velocity c.) However, a straightforward generalization of the above clock calculation yields[25] the expectation value for the time of flight over a distance L to be the physically reasonable relativistic generalization of Eq. (2),

$$T = L/v \tag{3}$$

$$v = pc/E \tag{4}$$

where $-c < v < +c$. The conclusion is that time *differences* can be sensibly calculated and measured in quantum mechanics (subject to the normal quantum uncertainty in the measurement of the clock pointer variable), whereas attempts to measure the *absolute* time of arrival of a particle at a point in space are doomed to failure. (There have been many attempts at defining an arrival time operator, all of which run into problems of one sort or another[26].)

3. TUNNELLING TIME

Additional apparent conflicts between quantum mechanics and relativity emerge when consideration is given to quantum tunnelling. Attempts to define tunnelling time have led to an extensive and confused literature[22,27]. Most theoretical treatments focus on the behaviour of a wave packet as it traverses a square barrier. However, such a barrier is dispersive, so the packet is disrupted by the experience. Also, interference between parts of the wave packet reflected from the barrier and parts still approaching further complicates matters.

A simple heuristic argument to estimate the tunnelling time goes as follows. To surmount a square barrier of height V, a particle with energy E must "borrow" an amount of energy $V - E$. According to the uncertainty principle, this must be "repaid" after a time $T = \hbar/(V - E)$. For a barrier of width L, the effective speed of the particle during the tunnelling process must exceed $L(V - E)/\hbar$. But now a puzzle emerges. As the height V of the potential hill is increased, so the tunnelling time decreases, i.e. the more repulsive the potential, the *faster* the particle moves. Because L has no upper bound, there seems to be no impediment to the velocity exceeding the speed of light for large L, in apparent violation of relativistic causality. Indeed, such claims have recently been made[28].

I shall now argue that an application of the Peres quantum clock model to measure the tunnelling time removes the apparent conflict with relativistic causality[29]. First consider a particle scattering from a potential step of height V situated at $x = 0$. The stationary state wave function has space-dependent part

$$e^{ikx} + Ae^{-ikx} \qquad x < 0, \tag{4}$$

$$Be^{-px} \qquad x > 0. \tag{5}$$

where $p = [2m(V - E)/\hbar^2]^{1/2}$. Suppose we require the expectation value for the time of flight of the particle to travel from $x = -b$ to the barrier and back again. The phase change is easily calculated to be $2kb + \alpha$, where $\alpha = \arctan(\mathrm{Im}\, A/\mathrm{Re}\, A)$ and $A = -(p + ik)/(p - ik)$. From this we obtain for the out-and-back time

$$T = 2b/v + 2m/kp = 2(b + d)/v \tag{6}$$

where $d \equiv 1/p$ is the penetration depth of the evanescent into the potential step. The term $2b/v$ is the time of flight from $x = -b$ to the potential step at $x = 0$ and back again, at the classical velocity v, whilst $2d/v$ represents the additional sojourn time in the classically forbidden region beneath the potential step. Thus the effective distance from $x = -b$ to the step is increased from b, the classical distance, to $b + d$. Note that if $E > V$, p is imaginary, A is real and $\alpha = 0$, and the transit time reduces to the classical result $2b/v$. The reflection from the step is instantaneous.

In the case of a square hill, V = constant > 0 in the interval $[0, L]$ and zero elsewhere, the wave function is

$$\begin{array}{ll} e^{ikx} + Ae^{-ikx} & x < 0 \\ Be^{px} + Ce^{-px} & 0 < x < L \\ De^{ikx} & x > L. \end{array} \tag{7}$$

The phase of the incident part of the wave function at $x = 0$ is 0. The phase of the emergent wave function at $x = L$ is given by the phase of De^{ikx}. Using continuity at $x = 0$ and $x = L$, the phase change is found to be

$$\theta(E) = \arctan\{[p^2 - k^2)/2kp]\tanh(pL)\} \tag{8}$$

yielding a corresponding time for tunnelling of

$$T = (2m/\hbar)\{k(p^2 - k^2)L + [(p^2 + k^2)^2/2kp]\sinh 2pL\}/[(p^2 + k^2)^2\cosh^2 pL - (p^2 - k^2)^2]. \tag{9}$$

The effect of the barrier width L on the particle's tunnelling speed may be extracted from Eq. (9). Generally, thin barriers slow the impinging particle down, but thick barriers speed them up. The effective velocity under the barrier for small L is approximately $2v/(2 + V/E) < v$, whereas for large L it approaches $LE^{1/2}(V - E)^{1/2}$. Note that the tunnelling time for large L approaches the constant value $\hbar[E(V - E)]^{-1/2}$. This is similar to the result found from the naive argument based on the uncertainty principle, but with the interesting difference that the "borrowing" requirement is not simply $V - E$, but the harmonic mean of this quantity and E. The effective tunnelling velocity exceeds the speed of light c when

$$pL > 2mc/k = 2(\text{de Broglie wavelength})/(\text{Compton wavelength}). \tag{10}$$

(Strictly, one should use a relativistic treatment to justify this condition[25].) Although for thick barriers the transmission probability is very small, it is nonzero, and we have to confront the consequences for causality if it is indeed the case that the occasional particle can tunnel faster than light.

A violation of causality will come about if observer A can send information to an observer B a distance L away such that it arrives before a time L/c has elapsed. Could A use an electron to encode this information, and arrange for it to tunnel through a barrier to B in the knowledge that, albeit only occasionally, B will get to receive the electron before a time L/c? The analysis given here suggests the answer is no. To achieve physical causality violation, A must be able to determine the moment of transmission of the information. But as we have seen, the model system discussed here can determine only the time *difference* between the moment of "transmission" and "reception" of the particle - not the absolute time of transmission. Any faster-than-light propagation would therefore be fortuitous - entirely random and uncontrollable. Quantum tunnelling may violate the spirit of relativity, but it does not seem to violate the letter.

4. THE PRINCIPLE OF EQUIVALENCE

The general theory of relativity is founded on the principle of equivalence of inertial and gravitational mass, a property normally associated with Galileo's experiment of dropping different masses from the leaning tower of Pisa. More famously, Galileo was able to deduce that all bodies accelerate equally under gravity using a thought experiment[30]. But consider, recalling my conversation with Wheeler, how Galileo would have constructed his thought experiment had he attempted to base it on quantum mechanics rather than classical mechanics (and, after all, quantum mechanics is supposed to be the more fundamental theory). If we imagine dropping two quantum particles of different masses from rest at a fixed height and at the same moment, then following their descent, we immediately run into the problem that by specifying the initial position we create uncertainty in the initial momentum. It follows that we cannot also specify the particles to be initially at rest. In Galileo's time it would have appeared incredible to contemplate actually performing such an experiment with atoms or subatomic particles. Today, however, it is possible to observe the motion of atoms and neutrons in the Earth's gravitational field[15-18].

Classically, when the inertial mass m_i and the gravitational mass m_g are equated, the mass drops out of Newton's equations of motion, implying that particles of different mass with the same initial conditions follow the same trajectories. But in Schrödinger's equation the masses do not cancel:

$$-(\hbar^2/2m_i)\partial^2\psi/\partial x^2 + m_g g x\psi = i\hbar\partial\psi/\partial t \qquad (11)$$

implying mass-dependant differences in motion. If the motion of a quantum particle is represented by a sharply-peaked wave packet, then by Ehrenfest's theorem one expects that the expectation value of the particle's position will follow a geodesic, and so provide a natural classical limit that complies with the equivalence principle. There will, however, be mass-dependant quantum fluctuations about the mean geodesic motion[31]. In experiments where the positions of the falling atoms are determined using laser scattering, a wave packet treatment is appropriate.

We may also consider quantum states of a very different form, however, for example energy eigenstates extended over a large region of space, or even entangled states. Such quantum states do not have classical counterparts in localised bodies moving on well-defined trajectories. Rather, they might correspond to a steady flux of particles coming from a great distance. What can be said about the principle of equivalence in such a case?

Consider a variant of the simple Galileo experiment, where particles of different mass are projected vertically in a uniform gravitational field with a given initial velocity v. Classically, the particles will return a time $2v/g$ later, after rising to a height $x_{max} = 2(2x/g)^{1/2}$. But quantum particles can tunnel into the classically forbidden region above x_{max}, to a distance that is mass-dependant. We might therefore expect a slight mass-dependant "quantum delay" in the return time, representing a clear breach of the equivalence principle. So what do we find when analysing the up-and-down travel time using a quantum clock[32]?

Equation (11) may be solved in terms of Airy functions Ai (Bessel functions of order 1/3). The space-dependant part of the stationary state solutions is

$$u(x) \propto \mathrm{Ai}\,[(x-b)/a] \qquad (12)$$

where

$$a = (\hbar^2/2m_i m_g g)^{1/3}, \qquad b = E/m_g g. \tag{13}$$

(Henceforth, unless specified explicitly, I shall put $m_i = m_g \equiv m$.) The wave function Ai decays exponentially in the classically forbidden region $x > b$ with a penetration depth ~ a. For an electron near the Earth's surface, this distance is about 1mm. Close to an object such as the International Space Station, the tunnelling distance would be several orders of magnitude greater.

The right hand side of Eq. (12) may be expressed as

$$\tfrac{1}{3}\sqrt{z}\{[e^{i\pi/3}J_{1/3}(\zeta) + e^{-i\pi/3}J_{-1/3}(\zeta)] + [(1 - e^{i\pi/3})J_{1/3}(\zeta) + (1 - e^{-i\pi/3})J_{-1/3}(\zeta)]\} \tag{14}$$

where

$$\zeta = \tfrac{2}{3}z^{3/2}, \qquad z \equiv (b - x)/a > 0. \tag{15}$$

The terms in the square brackets in Eq. (14) correspond to incident (up-moving) and reflected (down-moving) waves respectively, with currents $\pm (N^2/4\pi)(2\hbar g/m)^{1/3}$ = constant (N being a normalisation factor). In the large z limit

$$\mathrm{Ai}(-z) \approx \tfrac{1}{2}\pi^{-1/2}z^{-1/4}e^{i(\zeta-\pi/4)} + \tfrac{1}{2}\pi^{-1/2}z^{-1/4}e^{-i(\zeta-\pi/4)}$$

$$\approx \pi^{-1/2}z^{-1/4}\sin(\zeta + \pi/4) \tag{16}$$

which contains incident and reflected waves of equal amplitude but different phase.

The phase change between the incident and reflected waves at $x = -X < 0$, evaluated far from the classical turning point, is

$$\theta = 2(\zeta + \pi/4) = 4/3[(E/mg - X)/a]^{3/2} - \pi/4, \tag{17}$$

from which the Peres clock yields the up-and-down transit time

$$T = (2\hbar/mga)\ [(E/mg - X)/a]^{1/2} = 2\sqrt{(2d/g)}, \tag{18}$$

where I have put $d \equiv b - X$, the distance to the classical tuning point from the point of vertical projection. Significantly, Eq. (18) is the *same* as the classical result; indeed, it is remarkable that although this is a quantum mechanical result, $\hbar$ drops out of the final answer in the asymptotic region $X \to \infty$. This amazing cancellation is entirely a property of the uniform gravitational potential mgx. It does not occur, for example, with a potential step, or an exponential potential[32]. The conclusion is that by making careful use of a quantum clock and forsaking knowledge about the absolute time of passage, one finds that, *even for highly non-classical states* the principle of equivalence holds, subject to $m_i = m_g$, and the particle traversing a distance >> a.

There is of course the possibility that the weak principle of equivalence will fail at some sufficient level of accuracy. An early speculation of this sort was the suggestion that gravity might break CPT invariance, leading to a differential gravitational acceleration of matter and antimatter[14]. Another pointer comes from the experiments of Werner and his colleagues[33] using a neutron interferometer that suggested a discrepancy of about one per cent from $m_i = m_g$, although this was not confirmed in the subsequent work by Zeilinger and his colleagues using a somewhat different technique[16]. Allowing for departures from the principle of equivalence, $m_i \neq m_g$, Eq. (18) is modified to

$$T = 2\sqrt{(2d/g)}\sqrt{(m_i/m_g)}. \tag{19}$$

This expression is also identical to the classical result. Evidently quantum mechanics does not introduce any *additional* deviations from the principle of equivalence. So even if the Werner discrepancy turns out to be real, *it cannot be attributed to quantum mechanics as such.* It must arise from modifications to gravitational theory.

How can we reconcile the fact that the wave function is non-vanishing beyond the classical turning point $x = b$, with the result that the up-and-down travel time is no greater than the classical transit time? Suppose we compute the turnaround time at $x = b$. The result is

$$T = 4.3^{-1/6}\Gamma(⅔)\hbar/13\Gamma(4/3)mga \approx 0.5(\hbar/mg^2)^{1/3}, \tag{20}$$

showing the expected delay due to the particle tunnelling into the classically forbidden region. What seems to be happening is this. Though there is a finite probability that a given particle may tunnel into the region above the classical turning point and return late, there is also a finite probability that the particle may back-scatter off the gravitational potential before it reaches $x = b$. This is consistent with the fact that the wave function (hence probability density) dips prior to $x = b$. It would appear that, far from the turning point, these two effects exactly cancel, leading to neither a shortening nor a lengthening of the classical turnaround time due to quantum effects. The distance scale for this approximation is determined by the length a, which roughly corresponds to one de Broglie wavelength from the turning point. Within this distance there are significant quantum corrections to the turnaround time. For an electron near the Earth's surface, the tunnelling delay at $x = b$ is about 4 ms. For a space-based experiment, utilising the small gravitational field of the space vehicle, the delay would be very much greater.

5. DISCUSSION AND FUTURE WORK

The Peres clock is itself a quantum system, and so subject to intrinsic uncertainty in its performance. There will also be a back-reaction of the operation of the clock on the measured particle[24]. These effects introduce errors in ΔT comparable to Eq. (20). Some improvement may be achieved by replacing the smooth interaction Hamiltonian $P(x)\omega J$ by a series of kicks[34], which introduces a numerical advantage. Also, the back-reaction may be reduced by making the coupling between the clock and the particle arbitrarily small, but at the expense of increasing the uncertainty in the clock pointer position. However, the latter uncertainty may be compensated by introducing a large ensemble of identical systems, and regarding the time measurement as a *weak measurement*[35]. It is in this ensemble sense that the times computed in this paper are to be regarded.

I have restricted attention to the so-called weak equivalence principle. One might also enquire into the status of the strong or Einstein equivalence principles in quantum mechanics. Einstein made the postulate that all of physics in a uniform gravitational field should be locally equivalent to the physics in a uniformly accelerated frame. It is well-known that under a transformation of coordinates to an accelerated reference frame

$$\begin{aligned} x' &= x - vt - ½gt^2 \\ t' &= t \end{aligned} \tag{21}$$

the Schrödinger equation for a free particle is transformed to a Schrödinger equation for a particle moving in a uniform gravitational potential with g. So there is a formal correspondence between a uniform gravitational field and a uniform acceleration in the underlying quantum kinematics, just as there is in classical kinematics. However, in quantum mechanics the relationship between the state of the system and the dynamical evolution is much more subtle than in classical mechanics. In the case of states that represent localised wave packets, the equivalence of acceleration and gravitation goes through in a reasonably straightforward manner[31]. But what about the case of the non-localised energy eigenstates considered in this paper? Here the situation is more complicated. The stationary states $e^{ikx - iEt/\hbar}$ don't transform into Ai $[(x - b)/a]e^{-iEt/\hbar}$ under Eqs. (21). Rather, the Airy functions are complicated linear combinations of exponential solutions and their complex conjugates. This would not matter if the results of the analysis were linear in the wave function, as in the case for the behaviour of wave packets which are made up of linear combinations of plane waves. But it is not the case for a measurement of the transit time using the Peres clock, because the time interval depends on a measurement of the phase change, and the sum of the phases of a superposition of waves is generally not the same as the phase of the sum. To test the quantum correspondence between stationary and accelerated reference frames requires an analysis of *accelerated* quantum clocks. It is far from clear that all model quantum clocks will respond equally when accelerated.

My treatment of falling quantum bodies has been restricted to a uniform gravitational field, but in fact the experiments are sensitive enough to measure gravity gradient corrections. Taking into account deviations from the potential *mgx* would certainly introduce mass-dependant quantum corrections. This is no surprise, as the non-locality of the wave function enables the particle to "feel out" the spacetime curvature. A similar situation arises in the case of classical electrically charged particles. Their rate of fall is less than that of uncharged particles because the electric field is non-local, and so is excited by the spacetime curvature, i.e. photons are produced by the relative motion between the electric and gravitational fields, resulting in an energy loss that introduces a type of frictional back-reaction[36].

I have also restricted attention here to energy eigenstates. Interest attaches to highly non-classical states, such as superpositions. Peters et. al.[18] tested the principle of equivalence for superpositions of energy eigenstates of caesium atoms, though their method involved laser interrogation, implying wave packet states. Generalizing the Peres clock treatment to energy superpositions, one finds that the amplitude of the clock's wave function for the free-fall transit time is a superposition of the two amplitudes for each energy eigenstate, resulting in two peaks for the expectation value of the transit time, accompanied by an intermediate peak arising from the interference term. Of rather more interest would be to construct superpositions of different total *mass* but the *same* centre-of-mass energy. These occur as small relativistic corrections to the total mass arising from the excitation of an internal degree of freedom (e.g. a superposition of excited and ground states of an atom). Ahluwalia has studied how superpositions of different mass eigenstates leads to the gravitational inducement of neutrino oscillations[9].

6. CONCLUSION

The application of sensitive quantum techniques has opened up a new domain for testing the consistency of the foundations of physics, and in particular the conceptual basis of the theory of relativity. Although quantum mechanics and relativity are formulated in very different ways, attempts to demonstrate an out-and-out clash have so far not succeeded. At first sight, quantum entanglement seemed to threaten relativistic causality via Bell-inequality superluminal signalling. But on closer investigation, the inherent uncertainty attaching to quantum states provides just what is needed to prevent faster-than-light information transfer[37]. More recently, quantum tunnelling experiments have hinted at superluminal effects, but again, on closer inspection, it appears as if no causality violation actually occurs[38]. The results reported here may be seen as further evidence that quantum mechanics contrives, often in very subtle ways, to maintain the internal consistency of physics by conforming with the founding principles of the theory of relativity: the speed of light as a limit to information transfer, and the weak principle of equivalence. In both cases, however, these principles have to be re-cast with essential caveats. Superluminal propagation of quantum particles *can* occur, but not in such a way as to convey information. The motion of quantum particles in a uniform gravitational field *is* mass-dependant, but only within a tunnelling time's duration, or when it arises from the differential spreading of wave packets.

Combined together, these results point to a deeper level of conceptual consistency than has hitherto been appreciated. Somehow, the deeper structure of quantum mechanics rescues the theory of relativity from apparent violations in all the scenarios so far examined. The way to reveal this deeper level of consistency is to re-formulate the theory of relativity – and in general, our description of spacetime structure – starting with the framework of quantum principles. But this raises a very profound question. Galileo was able to deduce by his thought experiment that gravitational acceleration was independent of mass, without knowing the laws of mechanics, and without writing down a single equation. He achieved this feat by envisaging a heavy mass and a light mass tethered together, and asking whether the presence of the attached light mass assisted or impeded the fall of the heavy mass. If, following Aristotle, the heavy mass fell faster, then the light mass would lag behind and restrain the heavy mass by pulling upwards on the tether. But, considering the total system of both masses together, the total mass is greater than that of the heavy mass alone, so the system as a whole should fall faster. Clearly there is a contradiction, resolved only if the rate of fall is independent of the mass.

Recalling my conversation with Wheeler, one might now ask, would Galileo have been able to make such a deduction within the framework of quantum mechanics? Is there a corresponding "quantum Galileo thought experiment"? Could one deduce the result described by Eq. (18) without solving Schrödinger's equation or going through the analysis of the quantum clock? To consider a specific scenario, a hydrogen atom could play the role of the two masses, with the tether being the Coulomb attraction. Would the wave function of a free proton in an energy eigenstate suffer the same phase shift as the wave function of a proton that formed part of hydrogen atom having the same centre of mass energy? Could one devise an interference experiment, avoiding decoherence, that involved the de-ionisation and re-ionisation of such a proton in one arm of a vertical interferometer? If it turns out to be impossible to deduce a suitable quantum principle of equivalence by pure thought, without using Schrödinger's equation, then this would be a very troubling conclusion, for it would imply that classical mechanics possesses some deep property in relation to gravitation that is not contained within quantum mechanics (except in the trivial sense of the classical limit). It would mean that classical mechanics was somehow more fundamental than quantum mechanics (as Bohr believed), and would run counter to the prevailing view that quantum mechanics should be the basis for all of physics[39].

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