Celestial illusions and ancient astronomers – Aristarchus and Eratosthenes

Thomas V. Papathomas[i]

Laboratory of Vision Research & Department of Biomedical Engineering,
Rutgers University, Psychology Building, Piscataway, New Jersey, 08854

ABSTRACT

When the moon is half, one would expect that a line starting from the moon's center and being perpendicular to the "shadow diameter" would, if extended, go through the center of the light source, namely, the sun. It turns out that, when the sun is visible, this extended line appears to aim significantly above the sun, which is the essence of the "half-moon illusion". The explanation advanced here is that this is not an optical illusion; instead, it can be explained by the relative sizes and distances of the earth, moon, and sun, and it hinges on the fact that the sunrays are nearly parallel with respect to the earth-moon system. It turns out that the ancients knew and used this near-parallelism of the sunrays. Eratosthenes, for example, used a simple but ingenious scheme to obtain a good estimate of the earth's circumference. An interesting question is: How did the ancients arrive at the conclusion that the sunrays are nearly parallel? This was probably a corollary, based on the immense size of the sun and its huge distance from the earth, as estimated by, among others, Aristarchus of Samos by a brilliantly simple method.

Keywords: Moon illusion, half-moon illusion, apparent distance and size, parallelism, ancient astronomy, geometry

1. INTRODUCTION

The "half-moon illusion" is much less known than the widely known "moon illusion", in which the moon's size appears much larger near the horizon than at or near its zenith[1,2]. The ancients knew about the moon illusion[3]; as early as the second century A.D., Ptolemy, a Greek astronomer – who, incidentally, used advanced mathematical formulations to promote the erroneous theory of a geocentric system – put forth the idea that an object seen against "filled space" (as is the case when the moon is seen near the horizon, against trees, houses, etc.) appears to be further away than the same object viewed against a void space (as is the moon at its zenith), even if the viewing distance is the same[3]. In the 11th century, Al-Hazan, an Arab astronomer proposed that the explanation of the moon illusion is based on its apparent distance from the earth. This "apparent-distance" theory is presently the most prevalent explanation[4].

This paper deals with the "half-moon illusion". As the name implies, the half-moon illusion can be observed when we can see half of the moon's disk, i.e., at the first and third quarters of the lunar month. The illusion is illustrated in Figure 1.

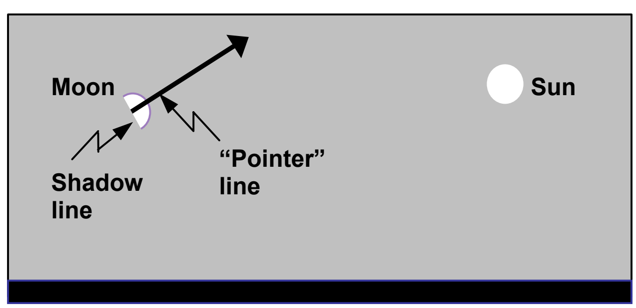

Figure 1. Schematic of the "half-moon illusion". The black bar on the bottom represents the earth and defines the horizon. The "pointer" line starts at the center of the moon, and is perpendicular to the shadow line. One would expect that the pointer line points to the sun, but it appears to point above the sun.

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[i] Correspondence: Email: papathom@rci.rutgers.edu; WWW: http://ruccs.rutgers.edu/~papathom/index.html; Telephone: 732-445-6533; Fax: 732-445-6715

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The "shadow diameter", or "shadow line", is the line that separates the illuminated from the dark part of the half moon. The "pointer line" is the line that starts from the moon center and is perpendicular to the shadow line. Since the sun is the light source, one would expect that the pointer line, if extended toward the sun, would pass through the sun's center. However, it appears that this line aims significantly higher than the sun[5,6], as you can see next time there is a half moon. B. Rogers pointed out this illusion - which he termed the "new moon illusion" - to S. Anstis[7], who discussed it, and gave a brief explanation that he attributed to M. Swanston. B. Schölkopf[8] has advanced an explanation based on the visual system's assumptions about the geometry of space. In this paper, I present a simple geometrical explanation based on the parallelism of the sunrays. Furthermore, I explore the question of how the ancients conceived the sunrays' parallelism, despite their convergence to the sun. The sunrays' parallelism was used heavily by ancient geometers. Most famously, Eratosthenes used it to get a rather accurate estimate of the earth's circumference in 230 B.C.

2. EXPLANATION BASED ON PARALLELISM OF SUNRAYS

The half-moon illusion can be explained based on the fact that sunrays are nearly parallel with respect to the earth-moon system. The explanation is schematized in Figure 2 for the special case in which the sun is just about to set at the end of the day, on the day of the moon's first quarter. The geometry is as viewed from a position in space above the earth's north pole, i.e., viewing the plane of the earth's elliptical orbit around the sun from above. Thus, the earth's silhouette is very close to the equator in Figure 2. The sunrays, shown by long arrows, fall nearly parallel on the earth and moon. When it is half-moon, the angle EMS between earth, moon, and sun is 90° (see dotted-dashed lines). At sunset, an observer on the earth (schematized by a "happy face") sees the sun down at the horizon, but *the apparent distance of the sun is comparable to that of the moon*. At sunset, the moon is straight overhead, and the pointer line appears to "point" to a position higher than the horizon, as shown in Figure 2. It is only because the sun's apparent distance is small that the illusion works; if we perceived the sun at infinity, then the pointer line *would* point to the sun. The insert icon shows what is perceived, following the same conventions as Figure 1. The explanation is similar, and can be generalized for observers who are at other locations on earth at this moment, in which the sun is above the horizon.

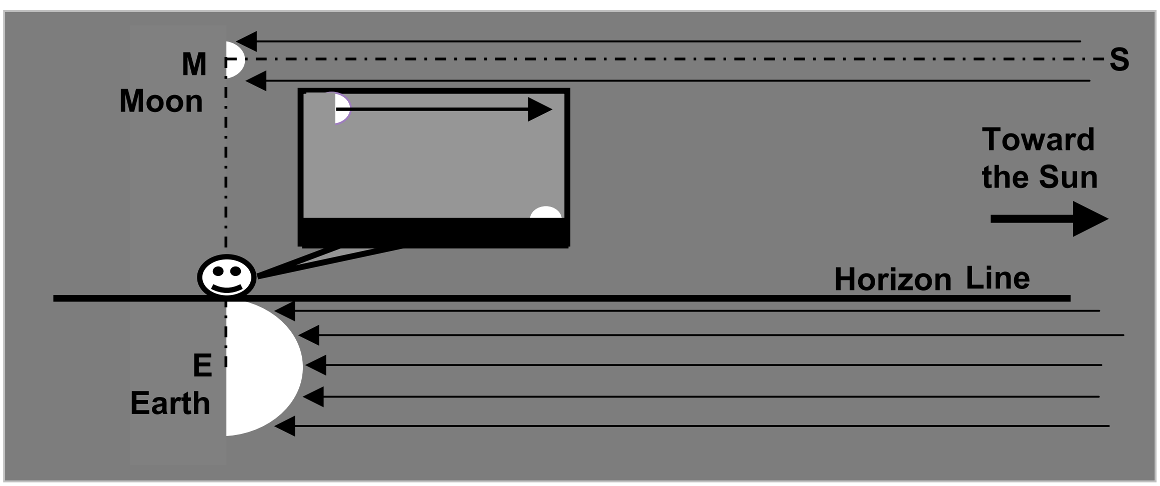

Figure 2. Schematic explanation of the "half-moon illusion". The geometry as viewed from a position in space above the earth's north pole. An observer on earth is shown by a "happy face", and his/her horizon line is indicated by a thick line. A schematic of what he/she sees is shown in the insert, using the same conventions as in Figure 1. See the text for details.

3. AN INTERESTING QUESTION – ANCIENT ASTRONOMY

How did the ancients infer that the sunrays are parallel? After all, in the real world the sunrays appear to converge to the sun disk, as can be seen in an open landscape on a cloudy day where the sun penetrates among the clouds. Figure 3 shows a similar scene with converging rays in an early foggy morning, which allows the sunrays to become clearly visible. It is hard to infer from such images that the sunrays are parallel. Nevertheless, ancient astronomers correctly considered that the sunrays are nearly parallel with respect to the earth-moon system, probably based on the relative sizes and distances of the earth, sun and moon (see subsection 3.2). In fact, Eratosthenes used the near-parallelism of the sunrays to obtain a rather accurate estimate of the earth's perimeter, as shown in Figure 4.

Figure 3. A scene in which the sunrays are clearly visible in the early morning fog. They appear to be far from parallel, yet we take it for granted that they are nearly parallel. How did the ancients arrive at the conclusion that they are nearly parallel?

3.1 Eratosthenes's estimate of earth's circumference. Eratosthenes[ii] started with two basic hypotheses (see Figure 4): (1) The earth is spherical; and (2) the sunrays are parallel. He knew that the sun was directly overhead the town of Syene, on the Tropic of Cancer (point Y in Figure 4, shown with a diamond symbol) at the summer solstice, at noon, because the sun reached the bottom of a perfectly vertical deep well (size-exaggerated as a black rectangle in Figure 4).

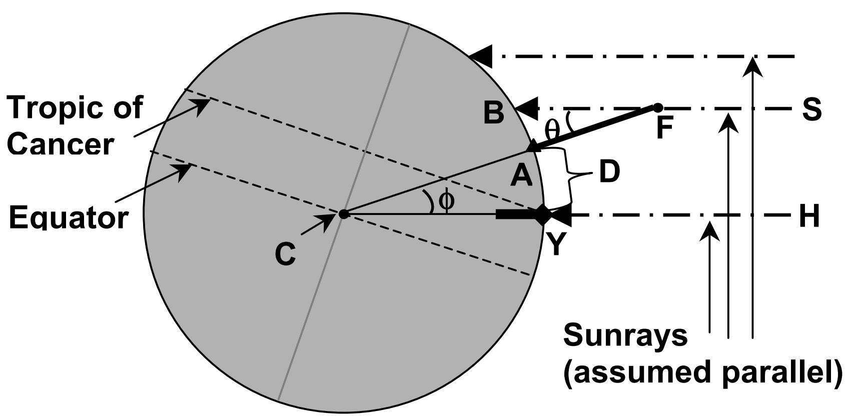

Figure 4. Eratosthenes's method of estimating the earth's circumference was critically based on the near-parallelism of the sunrays, shown here as dashed-dotted lines. See the text for details.

[ii] Eratosthenes was not only an accomplished astronomer and mathematician (of Eratosthenes's sieve fame). He was also an epic poet, geographer, historian, scientist, literary critic, and the chief librarian of Alexandria's celebrated library. He was called "beta" ("second best") and "pentathlos" because he was very good across many disciplines, but not the best in any one of them.

One such sunray, HY in Figure 4, being vertical to the earth's surface, must go through C, the earth's center. At noon, on the same day of the year, Eratosthenes made the following measurements in Alexandria (point A on Figure 4, shown by a triangle). Using a vertical stick of known height AF (size-exaggerated in Figure 4), he measured its shadow AB, and calculated θ, the angle AFB that the sunrays form with the local vertical FA, which also passes through C; Alexandria was selected because it lies almost on the same meridian as Syene. Now, since line FC intersects parallel sunrays SB and HY, angle θ is equal to angle ϕ, being alternate interior angles of parallel lines. Since the distance D between Alexandria and Syene is subtended by angle ϕ, the circumference of the earth P is given by $P = 360D/\phi$. The commonly quoted values measured by Eratosthenes are: θ=7.2°, D=5,000 stadia[9], where a stadium is about 166.7 meters. (This ancient unit of measurement corresponded to the length of an athletic stadium, and its value is not universally agreed upon). Thus, his estimate of P was 250,000 stadia = 41,675 km, a 4.4% error in overestimating the actual value of 39,919 km. Leaving aside practical problems (such as estimating the distance D, or knowing when is the precise moment of "noon"), this simple method yielded a truly accurate value of P!

3.2 Ancient estimates of sizes and distances for sun and moon. It is generally accepted that the ancients concluded that the sunrays are nearly parallel with respect to the earth-moon system, and that they arrived at this conclusion on the basis of the immense size of the sun, and its huge distance from the earth. But how was size and distance estimated? Aristarchus of Samos derived an estimate for the distance between the earth and the moon by measuring the size of the moon relative to the size of earth's umbra at the distance of the moon, during a lunar eclipse[10]; the method is too lengthy for the present paper. But Aristarchus's method of estimating the distance between the sun and the earth is ingenious in its simplicity, as illustrated in Figure 5.

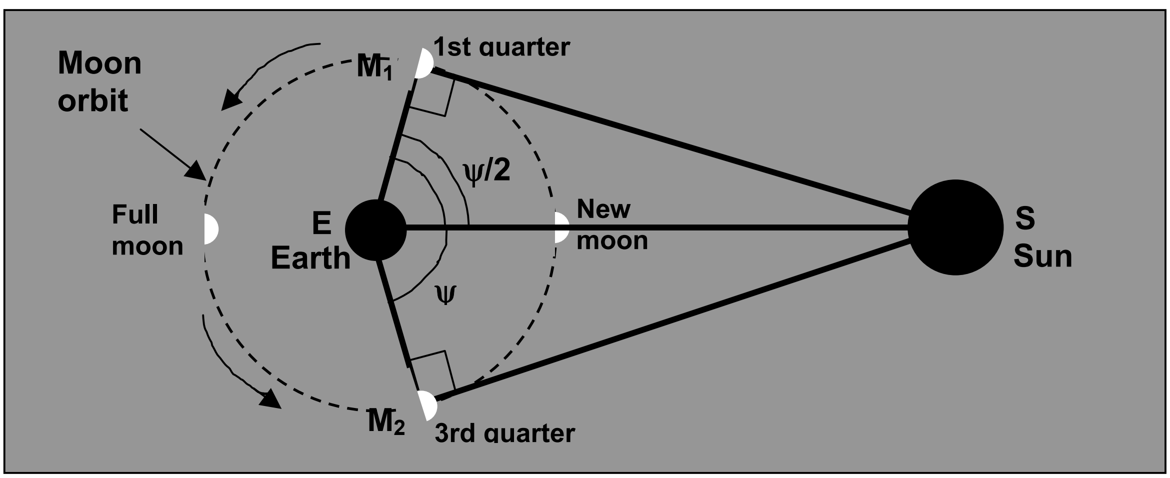

Figure 5. The ingeniously simple method used by Aristarchus of Samos to estimate the ratio $(SE)/(M_1E)$ of the sun-to-earth and the moon-to-earth distances. See the text for details.

His main assumption was that the moon's orbit around the earth is circular, which is a reasonable approximation. He then argued that the angle between the earth, the moon, and the sun at half-moon is a right angle; namely, during the first and third quarters, angle(EM_1S) = angle(EM_2S) = 90°, respectively. He then estimated the ratio $\psi/360°$ as the ratio of the time it takes for the moon to traverse the orbit from the third back to the first quarter to the total time for a full orbit. Once you know the angle ψ, then you can use trigonometry to find the ratio $r=(SE)/(ME)$ of the sun-to-earth and moon-to-earth distances. Aristarchus obtained an estimate of 90 for r, which is appreciably lower than the actual value of about 389[iii]. However, there are a few things arguing for his method: (1) In principle, the method is correct. (2) One difficulty

[iii] Eratosthenes, on the other hand, estimated the ratio $r=(SE)/(ME)$ to be about 1030, a bit higher than the actual value of 389. At least, they were both close to the reality, in concept. The author is not aware of the method used by Eratosthenes.

is in the enormous sensitivity of r as a function of the angle ψ: for example, when ψ=89°, r=57.3, whereas when ψ=89.5°, r=114.6. (3) To obtain an accurate value of ψ requires a good estimate for the time it takes for the moon to go from the third quarter back to the first quarter; this, in turn, requires a good judgment of the precise moment at which there is a half moon, emphasizing the need for good visual psychophysical methods (to my knowledge, we have no record of the details in Aristarchus's method). (4) The exact value of r is not as important as its magnitude: The ancients knew that the sun and the moon subtended the same visual angle, thus they also knew that the ratio of their diameters was proportional to r, the ratio of their distances from the earth. Thus, Aristarchus may not have known exactly how much bigger the sun was than the moon or the earth, but he knew that the sun was strikingly larger than either of them. Perhaps this knowledge of the immensity of the sun led Aristarchus to propose a heliocentric planetary system much earlier than Copernicus[11, 12]. (5) A corollary of the immensity of the sun and its enormous distance from the earth is the near parallelism of the sunrays, which, in turn, proved useful for other astronomical advances.

4. CONCLUSIONS - DISCUSSION

A simple explanation of the half-moon illusion has been proposed, based on the fact that the sunrays are nearly parallel as far as the earth-moon system is concerned. The illusion is obtained because we ascribe to the sun an apparent distance that is comparable to that of the moon, thus being much smaller than the veridical distance. A more complicated explanation was proposed by Schölkopf[8], who argues that far distant objects in space (such as the moon and the sun) are perceived as located on the celestial sphere, a dome-like surface, much like a planetarium, centered at the observer. Under this assumption, the straight sunrays that illuminate the moon would be perceived, if they were visible, as curved lines, thus explaining the moon's "tilt" (see Figure 1 and his Figure 2). He claims that his hypothesis can be tested in a mesoscopic environment, but the author is not aware of reports on such an experiment. Finally, the use of the near parallelism of sunrays in the explanation of this illusion raised the interesting question on how the ancients conceived of this parallelism, despite the clear apparent convergence, hence non-parallelism, of sunrays in natural images (e.g., Figure 3). Of course, all parallel lines appear to converge in the distance, after all: the sunrays *are* nearly parallel *and* seem to converge, and there is no contradiction in these statements. Nevertheless, the realization by the ancients that they are nearly parallel, a conclusion arrived at by simple geometrical reasoning, is worth marveling at.

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