Field Guide to

Probability, Random Processes, and Random Data Analysis

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Welcome to the *SPIE Field Guides*—a series of publications written directly for the practicing engineer or scientist. Many textbooks and professional reference books cover optical principles and techniques in depth. The aim of the *SPIE Field Guides* is to distill this information, providing readers with a handy desk or briefcase reference that provides basic, essential information about optical principles, techniques, or phenomena, including definitions and descriptions, key equations, illustrations, application examples, design considerations, and additional resources. A significant effort will be made to provide a consistent notation and style between volumes in the series.

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Field Guide to Probability, Random Processes, and Random Data Analysis

Developed in basic courses in engineering and science, mathematical theory usually involves deterministic phenomena. Such is the case for solving a differential equation that describes a linear system where both input and output are deterministic quantities. In practice, however, the input to a linear system, such as imaging or radar systems, can contain a "random" quantity that yields uncertainty about the output. Such systems must be treated by probabilistic rather than deterministic methods. For this reason, probability theory and random-process theory have become indispensable tools in the mathematical analysis of these kinds of engineering systems.

Topics included in this *Field Guide* are basic probability theory, random processes, random fields, and random data analysis. The analysis of random data is less well known than the other topics, particularly some of the tests for stationarity, periodicity, and normality.

Much of the material is condensed from the authors' earlier text *Mathematical Techniques for Engineers and Scientists* (SPIE Press, 2003). As is the case for other volumes in this series, it is assumed that the reader has some basic knowledge of the subject.

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Glossary of Symbols and Notation

$\mathbf{a}, \mathbf{x}, \mathbf{u}$, etc.	Random variable, process, or field
$B_{\mathbf{u}}(R)$	Autocovariance or covariance function of random field
$C_{\mathbf{x}}(\tau)$	Autocovariance or covariance function of random process
$C_{\mathbf{xy}}(\tau)$	Cross-covariance function
CDF	Cumulative distribution function
Cov	Covariance
$D_{\mathbf{x}}(\tau)$	Structure function
$E[.]$	Expectation operator
$E[g(\mathbf{x})\|A]$	Conditional expectation operator
$f_{\mathbf{x}}(x), f_{\mathbf{x}}(x,t)$	Probability density function
$f_{\mathbf{x}}(x\|A)$	Conditional probability density
$F_{\mathbf{x}}(x), F_{\mathbf{x}}(x,t)$	Cumulative distribution function
$F_{\mathbf{x}}(x\|A)$	Conditional cumulative distribution function
${}_pF_q$	Generalized hypergeometric function
$h(t)$	Impulse response function
$H(\omega)$	Transfer function
$I_p(x)$	Modified Bessel function of the first kind
$J_p(x)$	Bessel function of the first kind
$K_p(x)$	Modified Bessel function of the second kind
m, $m(t)$	Mean value
m_k	k'th standard statistical moment
$n!$	Factorial function
PDF	Probability density function
Pr	Probability
$Pr(B\|A)$	Conditional probability
PSD	Power spectral density
RV	Random variable
$R_{\mathbf{x}}(\tau)$	Autocorrelation or correlation function
$R_{\mathbf{xy}}(\tau)$	Cross-correlation function
$\mathfrak{R}_{\mathbf{x}}(\tau)$	Long-time-average correlation function
$S_{\mathbf{x}}(\omega), S_{\mathbf{u}}(\kappa)$	Power spectral density function
$U(x-a)$	Unit step function

Glossary of Symbols and Notation

Var	Variance
$\mathrm{Var}[\mathbf{x}\|A]$	Conditional variance
$\overline{\mathbf{x}(t)}$	Time average
z^*	Complex conjugate of z
$\gamma(c,x)$	Incomplete gamma function
$\Gamma(x)$	Gamma function
$\delta(x-a)$	Dirac delta function (impulse function)
μ_k	k'th central statistical moment
$\hat{\mu}(t)$	Estimator of mean value
$\sigma^2, \sigma_{\mathbf{x}}^2$	Variance
τ	Time difference $t_2 - t_1$
$\Phi_{\mathbf{x}}(s)$	Characteristic function
$\|\,\|$	Absolute value
$\in$	Belonging to
$\binom{a}{n}$	Binomial coefficient
$\langle\ \rangle$	Ensemble average
$\{\ \}$	Event
$\cap$	Intersection