

ELEMENTS OF
**Optical and Laser
Beam Scanning**

Modeling of Mirror and Prism Scanning Devices

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SPIE.

Dedicated to the memory of my mother, Professor Yujun Luo;
my father, Professor Hen Li; and my wife, Hui-Tse Li.

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Preface

Scanning technology for optical and laser systems is used in the controlled deflection of optical and laser beam for information transfer, such as actively or passively detecting events in a given direction (e.g., lidar), detecting information from a given surface (e.g., bar-code scanning) or inducing a physical effect (e.g., photoconductivity or photomagnetism) on the surface during a flying spot scan. All of these scanning systems have been applied to a number of useful products that have a direct bearing on our life.

The scope of this book is restricted to mirror scanning systems and rotating wedge prism scanning systems. Here, “mirror” means a plane reflecting surface, and “wedge” means a thin prism with a right triangle cross-section.

Over nearly 30 years, I have been interested in the mathematical analysis of scanning devices for optical and laser systems to yield results with higher accuracy than those obtained from geometrical construction of imaging of an object by a movable mirror or prism, which was a method to yield design data for scanning devices engineering when mathematical model is difficult to obtain from open publications, in which the exact form of important equations may be considered as proprietary.

My analytical results have been summarized according to the format of original research papers, most of which were written solely by me and published in archival journals. After their publication, I usually received emails from readers with questions, friendly comments and reprints requests. This situation was different from my experience acquired after publishing theoretical papers on diffraction and coherence of light in optical physics, although many of them have high citation numbers over a longer period of time. Reader reactions to my research encouraged me to prioritize this book as a summary of the results I obtained in the past.

I appreciated the opportunity that SPIE gave me to collect my published and unpublished manuscripts together, along with detailed commentary and corrections in a book so that readers need not search through old journals. Readers of this book are assumed to have a foundation in vector operation and calculus, and a reasonable knowledge of elementary optics and laser. Detailed proofs that require long derivations are sometimes omitted but can be found in either the appendices or the cited references.

This book is divided into three parts, starting with an introductory chapter for the laws of reflection and refraction and the mathematical preliminaries of analytical raytracing (most expressions are in vector forms). Chapters 2 through 4 are the first part, which covers topics of mirror scanning devices with one axis of rotation for conic-sections scanning. Chapters 5 through 6 are the second part, which covers topics of mirror scanning devices with two axes of rotation, e.g., gimballed mirror and Galvanometric scanners in cascade for two-dimensional scanning. Chapters 7 through 10 are in the third part to address Risley-prism-based beam-steering systems (i.e., rotating wedge prism scanning systems) with two to three elements for moving target searching and tracking.

Since each chapter in this book is an enlarged version of my technical paper focused on a specific topic as noticed by a single line about the main reference material of that topic at the beginning of each chapter, which may be helpful to readers who are interested in a specific scanning device and want to know its analytic model and computed results.

Writing a book is time-consuming and laborious; I doubt that many books have been written without some substantial help and encouragement by others. This book is no exception; therefore, I wish to express my appreciation to my friends at Symbol Technologies, Inc., especially Dr. Jerome Swartz and Dr. Satya Sharma.

I am obliged to SPIE Press, specifically, Dr. Eugene Arthurs, who initiated this project, and Mr. Scott McNeill, who helped bring it to fruition. I am also obliged to the Optical Society for permission to reproduce drawings from my papers published in the *Journal of Optical Society of America A* and *Applied Optics*. I thank my sister, Professor Xiaoyu Li; my niece, Lily Ji; and my friend, Earl O'Neil—without their encouragement and support, this book would not have been possible.

Yajun Li
October 2021

Acronyms and Abbreviations

AARBB	axis-aligned rectangular bounding box
AF	autocorrelation function
AR	aspect ratio
AR	coating anti-reflection coating
ATF	angular transfer function
AVC	angular vertical comb scanner
C_v	scanning spot velocity non-uniformity
CP1, CP2,...	notation for cusp
DA	deflection angle
DF	depth of scan field
DFM	dynamic focusing module
FOR	field of regard
FOV	field of view
G-C	Galvo-golf-club XY scanner
G-G	dual-Galvanometric XY scanner
G-P	Galvo-Paddle XY scanner
Galvo	Galvanometric scanner
HDTV	high-definition television
IC	integrated circuit
IRCM	infrared countermeasure
J&W	first-order formula for a Risley prism pair from the Book by Jenkins and White
LIN	scan linearity
LOS	line of sight
MEMS	micro-electro-mechanical system
MOEMS	micro-opto-electro-mechanical systems
N	scanned angular resolution
N_s	scanned linear resolution acquired from the motion of scanning spot along a translating distance S
OCT	optical coherence tomography
OXC	optical cross-connect
PD	prism director

PID	controller proportional–integral–derivative controller
RP	regular Risley prism
SCS	single crystal silicon
SDOF	single degree of freedom
SDTV	standard-definition television
SLB	scan line bow
SOI	silicon on insulator
TIR	total internal reflection