Laser Beam Propagation through Random Media

SECOND EDITION

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Contents

Preface to Second Edition

Since publication of the first edition of this text in 1998 there have been several new and important developments in the theory of beam wave propagation through a random medium that we have incorporated into this second edition. Also, there were some topics excluded in the first edition that are now included. Nonetheless, we recognize that the general field of wave propagation through random media has grown in the last several years beyond what we can adequately cover in this one volume. For that reason, the reader should not consider this text an exhaustive treatment of propagation through turbulence.

One specific change in notation introduced here is the use of σ_R^2 for the Rytov variance in place of σ_1^2 (except in Chap. 13) to avoid confusion of the latter with the scintillation index σ_I^2. Other changes/additions that now appear include the following:

- more worked examples and expanded sets of exercise problems
- models for the scintillation index under moderate-to-strong irradiance fluctuations
- models for aperture averaging based on *ABCD* ray matrices
- beam wander and its effects on scintillation
- theory of partial coherence of the source
- models of rough targets (other than Lambertian) for ladar applications
- phase fluctuations
- analysis of other beam shapes
- expanded analysis of free-space optical communication systems
- expanded imaging systems analysis

Foremost among the new theoretical developments is the *extension of the Rytov theory* from regimes of weak irradiance fluctuations into *moderate-to-strong fluctuation regimes.* Although much of this theory has been published in a companion text by the authors and C. Y. Hopen, called *Laser Beam Scintillation with Applications* (SPIE Press, 2001), we present it here in a somewhat more complete treatment along with the standard Rytov theory that formed the basis for the first edition. Another topic in this second edition concerns the effects of *beam wander* on the scintillation index associated with an untracked beam. Conventional theory predicts that the on-axis scintillation associated with a focused beam along a horizontal path and that for a collimated beam on an uplink path to space will experience a substantial reduction (by orders of magnitude) as transmitter beam size increases, provided there is limited beam wander. In the case of an untracked beam, however, the predicted reduction in scintillation will not occur.

Also included in this second edition is a treatment of *phase fluctuations*, incorporating the phase variance, structure function, covariance, and temporal power spectrum. Among other topics, we introduce models for and discuss the role of *partial coherence* (spatially) of the source beam in reducing scintillation—for example, in a free-space optical communication system. The same partial-coherence model can also be employed to describe the reflected radiation from a rough target like that which occurs in many laser radar applications.

In preparing this second edition, each chapter of the first edition was carefully examined for clarity and content, and most chapters have had some alteration; in such cases the material is either broadened or simply rearranged, or both. As a consequence, the second edition has expanded the original 12 chapters of the first edition into 18 chapters divided into three fundamental areas:

Part I: Basic Theory

1. Prologue—contains a brief discussion of fundamental concepts and application areas. It is basically the same as in the first edition, but now contains updated information on some of the application areas.
2. Random Processes and Random Fields—contains a brief introduction to random processes and random fields. Only minor changes appear from the first edition.
3. Optical Turbulence in the Atmosphere—introduces Kolmogorov theory and various spectral models. Only minor changes appear from the first edition.
4. Free-Space Propagation of Gaussian-Beam Waves—the introduction of higher-order Gaussian beam modes has been expanded from the first edition and we have also moved the free-space propagation through optical elements by the use of *ABCD* ray matrices to this chapter.
5. Classical Theory for Propagation Through Random Media—introduces the Rytov approximation and other basic theories of wave propagation through random media. The treatment of Rytov theory for *ABCD* optical systems now appears in this chapter as well as the extended version of the Rytov theory that permits its use in regimes of strong irradiance fluctuations.
6. Second-Order Statistics: Weak Fluctuation Theory—the second edition expands Chapter 6 from the first edition into Chapters 6 and 7. The discussion concerning the second-order field moment (mutual coherence function) is restricted to weak fluctuations but includes a new treatment of beam wander and slant path formulations in addition to the original horizontal path treatment.
7. Second-Order Statistics: Strong Fluctuation Theory—the parabolic equation method and extended Huygens-Fresnel principle are introduced as theories used for calculating the mutual coherence function under strong irradiance fluctuations. The method of effective beam parameters is also introduced for calculating the spatial coherence radius of a beam and the variance of beam wander displacements.

8. Fourth-Order Statistics: Weak Fluctuation Theory—the second edition expands Chapter 7 from the first edition into Chapters 8 and 9. Here we discuss scintillation models and the effect of beam wander on scintillation of both collimated and focused beams. Other new topics included here are a discussion of phase fluctuations and scintillation along a slant path.

9. Fourth-Order Statistics: Strong Fluctuation Theory—scintillation models for plane waves, spherical waves, and Gaussian-beam waves are separately developed based on the extended Rytov theory for the strong fluctuation regime. The gamma-gamma distribution for irradiance fluctuations is also introduced in this chapter, illustrating how the parameters of this model are completely determined by atmospheric conditions (refractive-index structure constant, inner scale, and outer scale).

10. Propagation Through Complex Paraxial *ABCD* Optical Systems—the propagation of a Gaussian beam wave through complex paraxial *ABCD* optical systems in the presence of atmospheric turbulence is featured here. In particular, we use the *ABCD* method to calculate the effect of a large-aperture receiver (aperture averaging) on the irradiance flux variance in the plane of a detector.

Part II: Applications

11. Free-Space Optical Communication Systems—here we examine the impact of scintillation on free-space optical communication systems that operate along a horizontal path. Various fade statistics are introduced, including the probability of fade and mean fade time.

12. Laser Satellite Communication Systems—we extend the treatment from Chapter 11 to examine laser satellite communication systems. Various second-order and fourth-order statistics are developed. Beam-wander-induced scintillation caused in an untracked uplink collimated beam is discussed in detail and several comparisons with recent simulation results are included.

13. Double-Passage Problems: Laser Radar Systems—the double-pass propagation problem associated with a laser radar system is treated here, which includes some new models developed since the first edition was published.

14. Imaging Systems Analysis—a brief treatment on performance measures of imaging systems is presented. Both coherent and incoherent systems are discussed. We also introduce the Zernike polynomials and related filter functions used in adaptive optics systems.

Part III: Related Topics

15. Propagation Through Random Phase Screens—the propagation of a beam wave through a random phase screen is taken up here, calculating the statistical quantities introduced in Chapters 6 and 8. The phase screen model also forms the basis for developing the (spatially) partial coherent beam analysis in Chapter 16.

16. Partially Coherent Beams—the notion of transmitter aperture averaging is presented for a partially coherent source and its impact on a free-space optical communication system. The same idea is used to model a rough target in a laser radar system.

17. Other Beam Shapes—here we examine a few effects of atmospheric turbulence on higher-order Gaussian beams and annular beam shapes.

18. Pulse Propagation—this chapter briefly covers some aspects (beam spreading and scintillation) on the propagation of ultrashort pulses.

The second edition contains three appendices at the end of the book: (I) a review of properties associated with some of the special functions; (II) a short table of integrals for easy reference purposes; and (III) tables of tractable formulas for the wave structure function, spatial coherence radius, and scintillation index as predicted by various theories and atmospheric spectrum models.

Last, we value the constructive comments made by several users of the first edition that helped to guide us in developing this second edition.

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Preface to First Edition

Laser beam propagation through random media is a subset of general wave propagation and scattering in a medium. By *random medium*, we mean one whose basic properties are random functions of space and time—common examples include the atmosphere, ocean, and biological media. The primary types of waves that are of general interest are electromagnetic (visible and radio) and acoustic. Hence, the field associated with a propagating wave may represent the transverse electric field in the case of electromagnetic radiation or it may represent the longitudinal pressure field in the case of an acoustic wave. The propagation of laser light, which is simply one form of electromagnetic radiation, is important in areas of application, such as weaponry, ranging, remote sensing, laser radar, mechanical positioning by laser beam, laser tracking, and distance measuring, among others. In addition, the scientific community has shown special interest for more than 30 years in the possibility of using high-data-rate optical transmitters for satellite communications. There are a number of advantages offered by optical wave systems over conventional radio frequency (RF) systems that can generally be attributed to the much shorter wavelengths associated with optical waves. However, the shorter wavelengths mean that optical waves are more susceptible to certain atmospheric effects.

The purpose of this text is to present an accessible account of wave propagation through random media with a particular emphasis on recent results concerning laser beam propagation through atmospheric turbulence. A complete theory of wave propagation through random media is not yet available—it remains an active area of research in many diverse fields like atmospheric optics, ocean acoustics, radio physics, and astronomy. However, the general theory is fairly well understood in certain asymptotic regimes and that is what we concentrate on here. The intended audience includes engineers and scientists who are interested in a sound understanding of propagation phenomena and the limitations imposed by the random medium on system performance. To this end, tractable analytic expressions are provided wherever feasible for a variety of statistical quantities affecting system performance. The book is structured in such a way that it may also serve as a graduate classroom text. To accommodate this latter category of potential readers, worked examples and exercise sets are provided at the end of most chapters.

Classical treatments of optical wave propagation are based primarily on the uniform plane wave and spherical wave models. Unfortunately, these simple wave models do not take into account various effects that can be directly attributed to the finite size of a beam wave as well as to its diverging and focusing

capabilities. The basic wave model that we use is the lowest-order Gaussian-beam wave, characteristic of a single transverse electromagnetic wave (TEM_{00}) emanating from a laser. By developing theory for a beam wave, we easily obtain as limiting cases many classical results for the infinite plane wave and spherical wave models. The general theory developed here applies to the optical (visible) and infrared (IR) segment of the electromagnetic spectrum, but may also be applicable to other electromagnetic radiation or even acoustic waves. In most cases we assume the random medium is the atmosphere for which small index-of-refraction fluctuations induced by random temperature variations are the main concern. We further limit our discussion to clear-air turbulence, i.e., the effects of scattering and absorption by aerosols or precipitation are ignored. Typical atmospheric inhomogeneities (scale sizes) are large compared with the wavelength of an optical/IR wave, so scattering of the wave is largely confined to the forward direction. This last condition permits use of the paraxial approximation that leads to the parabolic equation as the governing equation for the complex amplitude of the wave. For tractability reasons, the atmospheric models we utilize are based on assumptions of statistical homogeneity and isotropy.

Our approach to propagation problems involves the formulation of the lower-order field moments of the propagating wave from which important specializations are readily deduced. The multiple integrals that typically arise in characterizing these field moments are often formidable to evaluate analytically, and even when analytic results can be derived, they may be difficult to interpret owing to their complexity. For that reason, tractable approximations are usually presented for beam characteristics such as beam spreading, spatial coherence, angle of arrival, beam wander, and irradiance (intensity) fluctuations. Analytic solutions (including approximations) are generally preferred because they permit us to see how solutions depend on the important parameters of the problem. Pure numerical solutions are sometimes used in our analysis but these are inherently less revealing.

Chapter 1 contains a brief discussion of fundamental concepts and application areas involving optical wave propagation. The basic ideas of random processes, including the notions of structure function and spectral representation, are reviewed in Chapter 2; this chapter can easily be skipped for readers already versed in these topics. In Chapter 3, we review the classical Kolmogorov theory of turbulence (including temperature fluctuations and optical turbulence) and introduce several refractive-index power spectral models upon which subsequent analytic results are based. The single mode Gaussian-beam wave model in free space is discussed in Chapter 4, emphasizing the use of two sets of nondimensional beam parameters to characterize the optical wave at the transmitter and receiver. Although this departs from classical and contemporary works, we believe the consistent use of these nondimensional beam parameters throughout the text greatly assists the development of physical intuition for the reader. Higher-order Gaussian-beam modes are introduced in the last section of Chapter 4, but are only briefly discussed. The classical Born and Rytov weak fluctuation theories are presented in Chapter 5. Here we develop spectral representations for the first- and second-order complex phase perturbations that form the basis of all

statistical calculations in later developments that rely on the Rytov approximation. Chapters 6 and 7 are detailed treatments of line-of-sight propagation along horizontal paths using weak fluctuation theory. In these chapters, the general second- and fourth-order field moments are developed, from which a variety of important statistical quantities can be deduced. A parallel treatment is provided for the case of a random phase screen in Chapter 8. In Chapter 9, we extend the developments of Chapters 6 and 7 to nonhorizontal propagation path problems involving laser satellite communications. The concentration here is on fade statistics of the received signal for both uplink and downlink channels. The use of *ABCD* ray matrices to treat optical wave propagation through a system of cascaded optical elements is presented in Chapter 10. We consider in some detail the special case of a soft-aperture and thin lens combination, commonly called a "Gaussian lens," which includes a Fourier-transform-plane analysis and a simple imaging system. The ray-matrix method is sufficiently general that the random medium can be placed at arbitrary locations along the propagation path. The techniques introduced in Chapter 10 are then used in Chapter 11 to analyze double-pass propagation problems such as the geometry used in laser radar and lidar systems. The mean irradiance, spatial coherence radius, and scintillation index are discussed in detail for both monostatic and bistatic configurations involving a finite smooth reflector (e.g., a mirror or retroreflector). The special case of a finite diffuse (or Lambert) surface is also briefly discussed. In Chapter 12 we again consider line-of-sight propagation, but this time under strong fluctuation conditions. Some results based on the extended Huygens-Fresnel principle for calculating the mutual coherence function are presented here as well as the asymptotic theory for the scintillation index within the saturation regime.

The four appendices at the end of the book contain (I) a review of properties associated with some of the special functions of mathematics; (II) a short integral table appropriate for problems in this book; (III) tables of tractable formulas for the wave structure function, spatial coherence radius, and scintillation index as predicted by various theories and spectrum models; and (IV) a short collection of MATHEMATICA programs used in solving some of the worked examples. The MATHEMATICA programs in Appendix IV permit the reader flexibility in selecting wavelength, beam size, focus, propagation distance, and atmospheric conditions in solving a variety of propagation problems. These simple programs also serve as models from which the reader can easily develop additional programs. However, we should stress that neither knowledge of, nor access to, MATHEMATICA is required to use or understand the material in this book. The use of this software, or similar software, is optional.

Each chapter in the text is presented somewhat independent of the others so as to permit maximum flexibility for the reader. Consequently, some discussions are intentionally repetitious. All chapters end with a Summary and Discussion section that recalls the most important ideas and results from that chapter. References provided at the end of each chapter are not intended to be exhaustive. Rather, more recent references are usually cited along with general review papers or texts that contain surveys of many early references. With the exception of Chapter 12, the majority of the material is devoted to establishing results based

on weak fluctuation theory using the Rytov approximation. The reason for this is that many new analytic results based on this theory have been published in recent years but have not been summarized elsewhere. Moreover, there are important application areas like certain ground/space links involving laser satellite communications for which weak fluctuation theory may be applied.

We owe many thanks to Deborah Kelly and Ammar Al-Habash for working through most of the mathematics checking for errors. We also wish to extend our sincere appreciation to M. S. Belen'kii and W. B. Miller who served as reviewers. Their constructive criticism and useful suggestions contributed greatly to the final version presented here. Finally, we would like to thank our editor Eric Pepper for his support of this project and the production staff at SPIE for their help in preparing the book for publication.

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Symbols and Notation

A	Aperture averaging factor
$B_x(t_1, t_2)$, $B_x(\tau)$	Time correlation function of quantity x
$B_x(\mathbf{R}_1, \mathbf{R}_2)$, $B_x(\mathbf{R})$, $B_x(R)$	Spatial covariance function of quantity x
$B_I(\mathbf{r}_1, \mathbf{r}_2, L)$, $B_I(\rho, L)$	Covariance function of irradiance
$B_I(\tau, L)$	Temporal covariance function of irradiance
$B_I^{iR}(\mathbf{r}, L)$, $B_{\ln I}^{iR}(\mathbf{r}, L)$	Correlation functions associated with amplitude enhancement of reflected wave
$B_X(\rho)$, $B_Y(\rho)$	Large-scale and small-scale covariances
$B_{\ln X}(\rho)$, $B_{\ln Y}(\rho)$	Large-scale and small-scale log covariances
$b_I(\rho, L)$	Normalized covariance function of irradiance
BER	Bit error-rate
$C(\mathbf{r})$	Random amplification factor of irradiance
$C_I^{iR}(\mathbf{r}, L)$	Irradiance correlation
C_n^2, $C_n^2(h)$	Refractive-index structure parameter
C_T^2	Temperature structure parameter
CNR	Carrier-to-noise ratio
CTF	Coherent transfer function
c	Speed of light ($=3 \times 10^8$ m/s)
D_G	Diameter of Gaussian lens
$D(\mathbf{r}_1, \mathbf{r}_2, L)$, $D(\rho, L)$	Wave structure function
$D_n(R)$	Index of refraction structure function
$D_x(t_1, t_2)$, $D_x(\tau)$	Time structure function of quantity x
$D_x(\mathbf{R}_1, \mathbf{R}_2)$, $D_x(\mathbf{R})$, $D_x(\mathbf{R})$	Spatial structure function of quantity x
DOC	Modulus of the complex degree of coherence
$dv(\mathbf{K}, z)$	Random amplitude of index of refraction
erf(x), erfc(x)	Error functions
$E_n(\mathbf{r}_1, \mathbf{r}_2)$, $n = 1,2,3$	Second-order moments of the complex phase perturbations
EBS	Enhanced backscatter
EG	Equal gain coherent detection scheme
EO	Electro-optics
F	Phase front radius of curvature of beam at receiver
F_G	Effective focal length of Gaussian lens
F_T	Fade level (in dB) below the mean on-axis irradiance

F_0	Phase front radius of curvature of beam at transmitter
$_pF_q$	Generalized hypergeometric function
$F_n(\kappa_x, \kappa_y, 0; z)$	Two-dimensional spatial power spectrum of refractive index
FAR	False alarm rate
FSO	Free space optics (optical)
$G(\mathbf{S}, \mathbf{R})$, $G(\mathbf{s}, \mathbf{r}; L)$	Green's function
$G_X(\kappa, l_0, L_0)$, $G_X(\kappa)$, $G_Y(\kappa)$	Large-scale and small-scale filter functions
GOM	Geometrical optics method
$\hat{H}(\nu)$	Coherent transfer function (CTF)
$H_n(x)$	Hermite polynomial of degree n
$I^0(\mathbf{r}, L)$	Irradiance of beam in free space
$I(\mathbf{r}, L)$	Irradiance of beam in random medium
$I_\nu(x)$	Modified Bessel function of order ν
i_{IF}	Intermediate frequency (IF) signal current
i_S	Signal current in a detector
i_N	Shot noise current in a detector
$J_\nu(x)$	Bessel function of order ν
$\mathbf{K}$	Vector spatial wave number
$K_\nu(x)$	Modified Bessel function of order ν
k	Wave number of beam wave ($=2\pi/\lambda$)
L	Propagation path length
L_f	Distance from receiver lens to photodetector
$L_n^{(m)}(x)$	Associated Laguerre function of degree n
L_0	Outer scale of turbulence
l_0	Inner scale of turbulence
MCF	Mutual coherence function
MTF	Modulation transfer function
$n(\mathbf{R})$	Index of refraction
$n_1(\mathbf{R})$, $n_1(\mathbf{r}, z)$	Random fluctuation in index of refraction
OTF	Optical transfer function
$\mathbf{p}$	Transverse vector between two observation points
$\Pr_d$	Probability of detection
$\Pr_{fa}$	Probability of false alarm
$\Pr(I \leq I_T)$	Probability of fade below threshold I_T
P_S	Signal power
$p(L)$	Propagation parameter ($= \Theta_0 + i\Lambda_0$)
PDF	Probability density function
PSF	Point spread function
Q_m, Q_l	Nondimensional inner-scale parameter ($= L\kappa_m^2/k$, $L\kappa_l^2/k$)
Q_0	Nondimensional outer-scale parameter ($= L\kappa_0^2/k$)

$\mathbf{R}$	Position vector in three dimensions
$R_x(t_1, t_2)$, $R_x(\tau)$	Time correlation function of quantity x
$\mathbf{r}$	Transverse position of observation point
r_0	Atmospheric coherence width (Fried's parameter)
$S(\mathbf{r}, L)$	Random phase
$S_I(\omega)$	Power spectral density of irradiance
SNR	Signal-to-noise ratio
SR	Strehl ratio
$U(x)$	Unit step function
$U_0(\mathbf{R})$, $U_0(\mathbf{r}, z)$	Complex amplitude of the field in free space
$U(\mathbf{R})$, $U(\mathbf{r}, z)$	Complex amplitude of the field in random medium
$u(\mathbf{r}, z)$	Field of the propagating wave
W_0	Beam radius at transmitter
W	Beam radius in free space at receiver
W_B	Beam radius in free space at the waist
W_{LT}	Long-term beam radius in random medium at receiver
W_{ST}	Short-term beam radius in random medium at receiver
W_G	Radius of Gaussian lens
W_R	Radius of Gaussian target (reflector) surface
WSF	Wave structure function
α, β	Parameters of the gamma-gamma distribution
α_0	Complex parameter at transmitter $(= 2/kW_0^2 + i/F_0)$
α_G	Complex parameter at Gaussian lens $(= 2/kW_G^2 + i/F_G)$
α_R	Complex parameter at Gaussian target $(= 2/kW_R^2 + i/F_R)$
β_0^2	Rytov variance for a spherical wave
$\Gamma(x)$	Gamma function
$\Gamma_2(\mathbf{r}_1, \mathbf{r}_2, L)$	Mutual coherence function
$\Gamma_4(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4, L)$	Fourth-order moment of the field
γ	Propagation path amplitude parameter
$\delta(x - a)$, $\delta(\mathbf{K} - \mathbf{K}')$	Dirac delta function
η_X, η_Y	Nondimensional cutoff frequencies for filter functions
θ_0	Isoplanatic angle
Θ_0	Beam curvature parameter at transmitter
Θ	Beam curvature parameter of the beam at receiver
Θ_e, Λ_e	Effective beam parameters at receiver
κ	Scalar spatial wave number

κ_l	Inner-scale wave number parameter ($= 3.3/l_0$)
κ_m	Inner-scale wave number parameter ($= 5.92/l_0$)
κ_X, κ_Y	Large-scale and small-scale cutoff spatial frequencies for filter functions
Λ_0	Fresnel ratio of beam at transmitter
Λ	Fresnel ratio of beam at receiver
λ	Wavelength
ν_0	Quasi-frequency associated with the irradiance covariance function
ρ	Scalar separation between two observation points
ρ_0	Transverse spatial coherence radius
σ_R^2, σ_1^2	Rytov variance for a plane wave
σ_B^2	Rytov variance for a Gaussian-beam wave
σ_G^2	Rytov variance for a Gaussian-beam wave with inner scale
σ_I^2	Scintillation index (normalized irradiance variance)
$\sigma_I^2(D)$	Irradiance flux variance for a collecting aperture of diameter D
$\sigma_{I,l}^2$	Longitudinal component of scintillation index
$\sigma_{I,r}^2$	Radial component of scintillation index
σ_N^2	Total noise power in detector current
σ_{PL}^2	Rytov variance for a plane wave with inner scale
σ_{pe}^2	Pointing jitter variance (caused by beam wander)
$\sigma_S^2(\mathbf{r},L)$	Phase variance
σ_{SP}^2	Rytov variance for a spherical wave with inner scale
σ_X^2, σ_Y^2	Large-scale and small-scale scintillations
$\sigma_{\ln I}^2, \sigma_{\ln Ir}^2, \sigma_{\ln Il}^2$	Log-irradiance variances
$\sigma_{\ln X}^2, \sigma_{\ln Y}^2$	Large-scale and small-scale log-irradiance variances
σ_χ^2	Log-amplitude variance
$\Phi_n(\kappa)$	Three-dimensional spatial power spectrum of refractive index
$\chi(\mathbf{r}, L)$	Random log amplitude
$\psi_1(\mathbf{r}, L), \psi_2(\mathbf{r}, L)$	Complex phase perturbations of Rytov approximation
$\psi_1(\mathbf{r}, \mathbf{s}), \psi_2(\mathbf{r}, \mathbf{s})$	Complex phase perturbations of extended Huygens-Fresnel principle
Ω_f	Focusing parameter (geometric focus of beam)
Ω_G	Fresnel ratio characterizing radius of Gaussian lens

Ω_R	Fresnel ratio characterizing radius of Gaussian target (reflector)
$\langle \, \rangle$	Ensemble average