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Abstract. An identification of human eye retinas by applying the covariance function and wavelet theory is presented. The estimations of the autocovariance functions of the two digital images or single image are calculated according to random functions, based on the vectors created from the digital image pixels. The estimations of the pixel's vectors are calculated by spreading the pixel arrays of the digital images into single column. During the changing of the scale of the digital image, the wave frequencies of the colors of the single pixels are prekept, and the influence of the change of a scale in the procedures of the calculations of the covariance functions does not occur. The Red, Green, Blue (RGB) color model of the colors spectrum for the encoding of the digital images was applied. The influence of the RGB spectrum components and the tensor of colors on the estimations of the covariance functions were analyzed. The identity of the digital images is estimated by analysis of the changes of the correlation coefficient values in the corresponding diapason. [DOI: 10.1117/1.OE.52.7.073106]

Subject terms: retinal images; image processing; covariance function.

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1 Introduction

A number of papers are contributing to retinal imaging and image analysis.[1,2] Retinal researchers and practicians make more and more wide use of the digital images, which causes the need to improve the image processing algorithms. In this paper we stress on an identification of the digital images by applying the photogrammetric methods and random functions theory. The spatial positions of the digital image pixels are defined by the spatial region of the frequencies of color waves, i.e., by radiometric level, applying the Red, Green, Blue (RGB) coding format of the colors spectrum. The theoretical model is based on the stationary random function taking into account that the errors of the color wave frequencies are random and that they are of the same accuracy, i.e., a mean of errors is $M\Delta = \text{const} = 0$ and their dispersion is $D\Delta = \text{const}$, and that the covariance function of the digital images depends only on the difference of the arguments, i.e., on the pixel quantize interval. The estimations of the covariance function of the two digital images or the autocovariance function of the single image are calculated according to random functions, based on the vectors, created from the digital image pixels. The Fourier transformation[3,4] or wavelet theory[5–8] is used for processing the digital signals. The main goal of this article is to provide the opportunity for a continuous improvement of the core algorithms, driven by performance of the covariance analysis approach. Such algorithms could be used in various areas of research and practice, including public and clinical health, biomedicine, security systems, etc.

2 Covariance Model of the Light Frequencies Spectrum

Let us analyze the autocovariance and covariance theoretical models of the white light spectrum, which is combination of the colors of different frequencies. The various color systems [RGB, hue, saturation and value (HSV), luma information, in-phase, quadrature (YIQ), hue, lightness and saturation (HLS), etc.] are used to define the digital images. The RGB system is used widely. The image of this system can be easily redone to the image of the other color systems, in which one of the component will be light signal and the other components define the color. It is used in the visualization, because we know that the human eye retina is more sensitive to the changes of the light strength than to the changes of the color itself.

Let us apply a linear expression of the harmonic oscillations equation to describe the light frequencies spectrum:

$$a(t) = A\,\sin(\varphi + \varphi_0) = A\left(\sin\bar{\varphi} + \delta\varphi\cos\bar{\varphi} - \frac{1}{2}\delta^2\varphi\sin\bar{\varphi} + \dots\right), \tag{1}$$

where A is an amplitude of the frequencies, $\varphi = \omega t$ is a phase, φ_0 is an initial phase, $\omega = 2\pi f$ is a cycle frequency, f is a frequency, t is a time moment, $M\varphi = \bar{\varphi}$ is a mean of a phase, $\delta\varphi = \varphi - \bar{\varphi}$ is the random error of a phase.

In real world and in practice, the monochromatic (single fixed-frequency) oscillations do not exist. So, analyzing a single color from the RGB color system, we understand that it is a mix of single color frequencies in the very narrow interval of frequencies $\Delta\omega$.

We will use the first-order members of Eq. (1) in the calculations of the covariance expressions, because an influence of the higher-order members is negligible. The dispersion Da_i and, certainly, covariance $K(a_i, a_i)$ of the monochromatic oscillations of frequency ω_i has expression

$$Da_i = K(a_i, a_i) = M(a_i - Ma_i)^2 = M(A_i^2 \delta^2 \varphi_i \cos^2 \bar{\varphi}_i) = A_i^2 \cos^2 \bar{\varphi}_i \sigma^2 \varphi_i, \tag{2}$$

where $Ma_i = \sin \bar{\varphi}_i$, $M\varphi_i = \bar{\varphi}_i$ is the symbol of a mean, σ_{φ_i} is the standard deviation.

The coefficient of correlation of the same frequency is equal to

$$r(a_i, a_i) = \frac{K(a_i, a_i)}{\sigma_{a_i} \cdot \sigma_{a_i}} = 1. \tag{3}$$

The covariance of the two fixed different frequencies looks like

$$K(a_i, a_j) = M(\delta a_i \cdot \delta a_j) = M(A_i \cdot A_j \delta\varphi_i \times \delta\varphi_j \cos \bar{\varphi}_i \cdot \cos \bar{\varphi}_j) = A_i A_j \cos \bar{\varphi}_i \cdot \cos \bar{\varphi}_j M(\delta\varphi_i \cdot \delta\varphi_j) = 0, \tag{4}$$

where $\delta a_i = a_i - Ma_i$, $M(\delta\varphi_i \cdot \delta\varphi_j) = M\delta\varphi_i \cdot M\delta\varphi_j = 0$, because $M\delta\varphi_i = 0$ is a mean of the independent random errors.

The coefficient of correlation of the two fixed different-frequency oscillations, applying Eq. (4), is equal to

$$r(a_i, a_j) = \frac{K(a_i, a_j)}{\sigma_{a_i} \cdot \sigma_{a_j}} = 0. \tag{5}$$

Because in nature and in practice the fixed-frequency oscillations do not exist, so for the analysis of the covariance of the light spectrum colors, we will use the signal compositions, applying their interference. The main summing equation of the interference of two frequencies a_{ij} could be written as follows:[9]

$$a_{ij} = a_i + a_j + 2\sqrt{a_i \cdot a_j}\gamma \cos \Delta\varphi_{ij}, \tag{6}$$

where γ is a coefficient of the frequency coherence and $\Delta\varphi_{ij} = \varphi_i - \varphi_j$ is the difference of the oscillation phases. In further calculations we will adopt $A_i = 1$ and $\gamma = 1$, because the influence of the permanent multipliers do not come into play during determination of the correlation coefficients.

So we have such a linear expression of a sum of two different frequency interferences.

$$a_{ij} = \sin \bar{\varphi}_i + \sin \bar{\varphi}_j + 2\sqrt{\sin \bar{\varphi}_i \cdot \sin \bar{\varphi}_j} \cos \Delta\bar{\varphi}_{ij} + \cos \bar{\varphi}_i \delta\varphi_i + \cos \bar{\varphi}_j \delta\varphi_j + (\sin \bar{\varphi}_i \cdot \sin \bar{\varphi}_j)^{-1/2} \cos \Delta\bar{\varphi} \times (\sin \bar{\varphi}_j \cos \bar{\varphi}_i \delta\varphi_i + \sin \bar{\varphi}_i \cos \bar{\varphi}_j \delta\varphi_j) + 2\sqrt{\sin \bar{\varphi}_i \cdot \sin \bar{\varphi}_j}(-\sin \Delta\bar{\varphi}_{ij} \delta\varphi_i + \sin \Delta\bar{\varphi}_{ij} \delta\varphi_j), \tag{7}$$

where $Ma_{ij} = \sin \bar{\varphi}_i + \sin \bar{\varphi}_j + 2\sqrt{\sin \bar{\varphi}_i \cdot \sin \bar{\varphi}_j} \cdot \gamma \cdot \cos \Delta\bar{\varphi}_{ij}$.

An expression of the random error of the oscillations summing interference will be

$$\delta a_{ij} = \cos \bar{\varphi}_i \delta\varphi_i + \cos \bar{\varphi}_j \delta\varphi_j + (\sin \bar{\varphi}_i \cdot \sin \bar{\varphi}_j)^{-1/2} \cos \Delta\bar{\varphi}_{ij} \times (\sin \bar{\varphi}_j \cos \varphi_i \delta\varphi_i + \sin \bar{\varphi}_i \cos \bar{\varphi}_j \delta\varphi_j) + 2\sqrt{\sin \bar{\varphi}_i \cdot \sin \bar{\varphi}_j}(-\sin \Delta\bar{\varphi}_{ij} \delta\varphi_i + \sin \Delta\bar{\varphi}_{ij} \delta\varphi_j). \tag{8}$$

We can ignore the last member of Eq. (8), because its influence is not big and its value $2\sqrt{\sin \bar{\varphi}_i \cdot \sin \bar{\varphi}_j}(-\sin \Delta\bar{\varphi}_{ij} \delta\varphi_i + \sin \Delta\bar{\varphi}_{ij} \delta\varphi_j) \to 0$, when $\delta\varphi_i \approx \delta\varphi_j$.

The covariance expression of the oscillations summing interference could be written as follows:

$$K(a_{ij}, a_{ik}) = M(\delta a_{ij} \cdot \delta a_{ik}) = \cos^2 \bar{\varphi}_i \sigma_{\varphi_i}^2 + (\sin \bar{\varphi}_i \cdot \sin \bar{\varphi}_j)^{-1/2} \cos \Delta\bar{\varphi}_{ij} \cdot \sin \bar{\varphi}_j \cos^2 \bar{\varphi}_i \sigma_{\varphi_i}^2 + (\sin \bar{\varphi}_i \cdot \sin \bar{\varphi}_k)^{-1/2} \sin \bar{\varphi}_k \cos \Delta\bar{\varphi}_{ik} \cos^2 \bar{\varphi}_i \sigma_{\varphi_i}^2 + \sin^{-1} \bar{\varphi}_i (\sin \bar{\varphi}_j \cdot \sin \bar{\varphi}_k)^{-1/2} \cos \Delta\bar{\varphi}_{ij} \cos \Delta\bar{\varphi}_{ik} \times \sin \bar{\varphi}_j \sin \bar{\varphi}_k \cos^2 \varphi_i \sigma_{\varphi_i}^2, \tag{9}$$

where $M\delta\varphi_i = 0$, $M(\delta\varphi_i \cdot \delta\varphi_j) = M(\delta\varphi_i \cdot \delta\varphi_k) = M(\delta\varphi_j \cdot \delta\varphi_k) = 0$ because of the independent random errors multiplication product.

Further, Eq. (9) has the following expression:

$$K(a_{ij}, a_{ik}) = [1 + (\sin \bar{\varphi}_i \cdot \sin \bar{\varphi}_j)^{-1/2} \sin \varphi_j \cos \Delta\bar{\varphi}_{ij}] \times [1 + (\sin \bar{\varphi}_i \cdot \sin \bar{\varphi}_k)^{-1/2} \sin \varphi_k \cos \Delta\bar{\varphi}_{ik}] \cos^2 \bar{\varphi}_i \sigma_{\varphi_i}^2. \tag{10}$$

By using the similar calculations, we could write the formulae of the dispersion of the oscillations summing interference:

$$Da_{ij} = \sigma_{a_{ij}}^2 = [1 + (\sin \bar{\varphi}_i \cdot \sin \bar{\varphi}_j)^{-1/2} \sin \bar{\varphi}_j \cos \Delta\bar{\varphi}_{ij}]^2 \times \cos^2 \bar{\varphi}_i \sigma_{\varphi_i}^2 + [1 + (\sin \bar{\varphi}_i \cdot \sin \bar{\varphi}_j)^{-1/2} \sin \bar{\varphi}_i \cos \Delta\bar{\varphi}_{ij}]^2 \times \cos^2 \bar{\varphi}_j \sigma_{\varphi_j}^2. \tag{11}$$

The formulae of the correlation coefficient of the two oscillations summing interference:

$$r(a_{ij}, a_{ik}) = \frac{K(a_{ij}, a_{ik})}{\sigma_{a_{ij}} \cdot \sigma_{a_{ik}}}. \tag{12}$$

In further calculations we will adopt $\sigma_{\varphi_i} = \sigma_{\varphi_j} = \sigma_{\varphi_k} = \sigma_\varphi$.

The accuracy of the phase measurements of the white light colors is equable, i.e., the standard deviations of the oscillation phases of the discrete frequencies are equal, $\sigma_{\varphi_i} = \ldots = \sigma_{\varphi_k} = \sigma_\varphi$. It comes from the assumption that the distributions of the random errors of the spectrum components are asymptotically similar.

Equations (3), (5), and (12) show that correlation between oscillations of different frequencies does not exist, whereas it exists between oscillations of similar frequencies. The correlation between the compound oscillations of mixed frequencies exists in the case when these oscillations have the components of similar frequencies.

3 Model of the Covariance Functions of the Digital Images

To develop the theoretical model we will apply the presumption that the errors of the digital image pixel parameters are random. The random function is constructed by spreading the arrow of the digital image pixels according to columns into one-dimensional space along the same coordinate axis. In each column of the pixel arrays the trend of the corresponding column is eliminated. The parameters are the indexes of the pixel color intensities in the RGB format color spectrum. We will accept the random function constructed in such a way as a stationary function (in wide understanding), i.e., its mean $M[\varphi(t)] \to \text{const}$ and a covariance function depend on the argument difference only, $K_\varphi(\tau)$. The pixel arrows of the two segments of the single digital image or segments of the two images $h_l(u)$ ir $h_j(u+\tau)$, accepted as the realizations of the random functions, continuous covariance function $K_h(\tau)$ could be written as follows:[10,11]

$$K_h(\tau) = \frac{1}{T-\tau}\int_0^{T-\tau} \delta h_l(u)\delta h_j(u+\tau)\mathrm{d}u, \tag{13}$$

where $\delta h_l(u)$, $\delta h_j(u+\tau)$ is the centered pixel parameters segments, u is a parameter of the segment pixel, T is the length of the segment in conventional units, $\tau = k \cdot \Delta$ is a changeable quantise interval, Δ is a value of the pixel parameter, k is the number of pixels in the quantise interval.

The covariance function $K'_h(\tau)$ based on the measurement results could be estimated according to the following formulae:

$$K'_h(\tau) = K'_h(k) = \frac{1}{n-k}\sum_{i=1}^{n-k} \delta h_l(u_i)\delta h_j(u_{i+k}), \tag{14}$$

where n is the total number of discrete intervals.

Equation (14) could be applied in the form of the autocovariance and intercovariance function. In the case of the autocovariance function, the segments $h_l(u)$ and $h_j(u+\tau)$ are parts of the single digital image, and in the case of intercovariance function, these segments are parts of the two different images.

The estimation of the normalized covariance function is

$$R'_h(k) = \frac{K'_h(k)}{K'_h(0)} = \frac{K'_h(k)}{\sigma'^2_h}, \tag{15}$$

where σ'_h is an estimation of the standard deviation of the random function.

To eliminate the trend in the i column of the image pixels array, we can use the formula

$$\delta H_i = H_i - e \cdot \bar{h}_i^T = (\delta h_{i1}, \delta h_{i2}, \ldots, \delta h_{im}), \tag{16}$$

where δH_i is an array of reduced pixels of the i digital image, in which column trend was eliminated; H_i is an array of the pixels of the i image, e is a unit vector, dimension of which is $(n \times 1)$; n is the number of rows of the i array, $\bar{h}_i$ is a vector of the means of the i pixels array column, δh_{ij} is a j column (vector) of the reduced pixels of the i array.

The vector of the means of the columns of the i pixels array could be calculated according to

$$\bar{h}_i^T = \frac{1}{n} e^T \cdot H_i \tag{17}$$

or

$$\bar{h}_i = \frac{1}{n} H_i^T \cdot e. \tag{18}$$

The realization of the random function of the i pixels array of the digital image in the form of vectors has an expression

$$\delta h_i = \begin{pmatrix} \delta h_{i1} \\ \delta h_{i2} \\ \ldots \\ \delta h_{im} \end{pmatrix} = (\delta h_{i1}^T \delta h_{i2}^T \ldots \delta h_{im}^T)^T. \tag{19}$$

An estimation of the autocovariance matrix of the i pixels array of the digital image looks like

$$K'(\delta H_i) = \frac{1}{n-1}\delta H_i^T \delta H_i. \tag{20}$$

An estimation of the covariance matrix of the two digital images or two pixels arrays of the single digital image could be written as follows:

$$K'(\delta H_i, \delta H_j) = \frac{1}{n-1}\delta H_i^T \delta H_j, \tag{21}$$

where the dimensions of the arrays δH_i, δH_j should be equal.

Applying the theory of covariance functions, the influence of the RGB format color spectrum components on the expressions of the covariance functions of the digital images was analyzed. Also, the expressions of the covariance functions of the digital images were estimated using the RGB colors continuous spectrum in the sense of the color tensor.

The estimations of the covariance matrixes $K'(\delta H_i)$ and $K'(\delta H_i, \delta H_j)$ are reduced to the estimations of the correlation coefficient matrixes $R'(\delta H_i)$ and $R'(\delta H_i, \delta H_j)$.[10,11]

$$R'(\delta H_i) = D_i^{-1/2} K'(\delta H_i) D_i^{-1/2}, \tag{22}$$

$$R'(\delta H_i, \delta H_j) = D_{ij}^{-1/2} K'(\delta H_i, \delta H_j) D_{ij}^{-1/2}, \tag{23}$$

where D_i, D_{ij} are the diagonal matrixes of the main diagonal members of the corresponding covariance matrix estimations $K'(\delta H_i)$ and $K'(\delta H_i, \delta H_j)$.

4 Results of the Experiment and Analysis

The digital images of the human eye [right oculus dexter (OD) and left oculus sinister (OS)] retinas were used in the analysis. The digital images were taken by ordinary photo camera and coded in JPEG format. Example of the OD retina's digital image is presented in Fig. 1. The

Fig. 1 A digital image of an OD retina.

calculations were executed by computer procedures written in MATLAB.

The results of the calculations are presented in Table 1 and in Figs. 2 through 9. In the calculations of the covariance functions, the values of quantise interval were changed from 1 pixel till $n/2$ pixels (in our case, $n = 120000$, average number of pixels in the segment of a digital image). An analysis was done applying all RGB color tensors and its components—red R, green G, and blue B colors.

In Table 1 the summarized mean values of the correlation coefficients between the pixels arrays of the digital images of the right OD and left OS eye retinas are shown. The calculations were executed along the single RGB color spectrum components and according to RGB spectrum tensor.

In the calculations of the normalized covariance functions, the results are presented only for red color of RGB spectrum, because the changes of the results for other spectrum colors are negligible.

From the data in Table 1 we can see that the values of the correlation coefficients between OD and OS retina arrays are near zero along all RGB spectrum colors, except red color, where correlation coefficients between OD and OS pixel columns are $r_{OD} = 0.54$ and $r_{OS} = 0.40$. This means that retinas absorb (accept) all the color oscillations at the same level, with an exception R color. The values of the correlation coefficients along all the RGB color tensors are near one and this means there is a very good relation between both retinas along all the RGB frequencies complex.

The change $(-0.3 < r_2 < 1.0)$ of the correlation coefficients between the five columns of pixel arrays of the digital image of the OD retina is shown in Fig. 2, the change $(-0.4 < r_2 < 1.0)$ of the correlation coefficients between the five columns of pixel arrays of the digital image of the OS retina is shown in Fig. 3, and the change $(-0.7 < r_4 < 0.3)$ of the correlation coefficients between the five columns of pixel arrays of the digital images of the OD and OS retinas is presented in Fig. 4.

A normalized autocovariance function of the digital image of the retina OD is presented in Fig. 5. It describes a change of the correlation due to changes of the quantise interval k between pixels. The values of the correlation coefficient are decreasing from $r = 1.0$ at $k = 0$ to $r < 0, 1$ when

Table 1 The mean values of the correlation coefficients between the pixels arrays of the digital images.

Combinations of the arrays of the digital images of the right OD and left OS eyes	Mean value of the correlation coefficient			
	Red R	Green G	Blue B	RGB tensor
Between OD pixel columns	0.54	−0.15	−0.17	0.98
Between OS pixel columns	0.40	0.13	−0.11	0.98
Between OD and OS pixel arrays	−0.42	−0.097	0.11	0.95

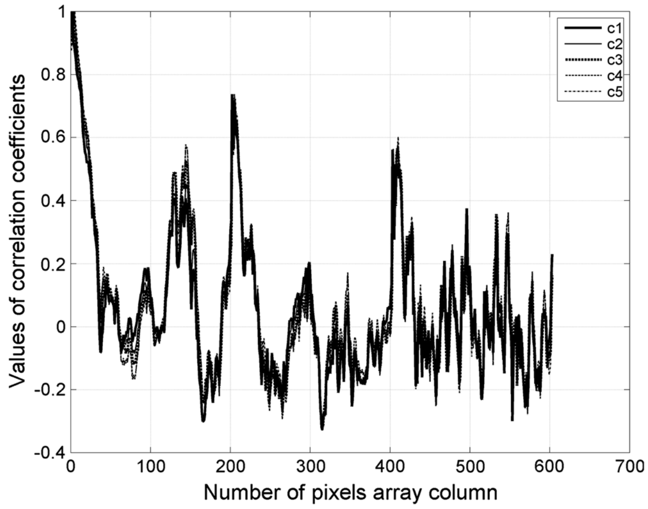

Fig. 2 The changes of the correlation coefficients between the five columns of pixel arrays of the digital image of the OD retina.

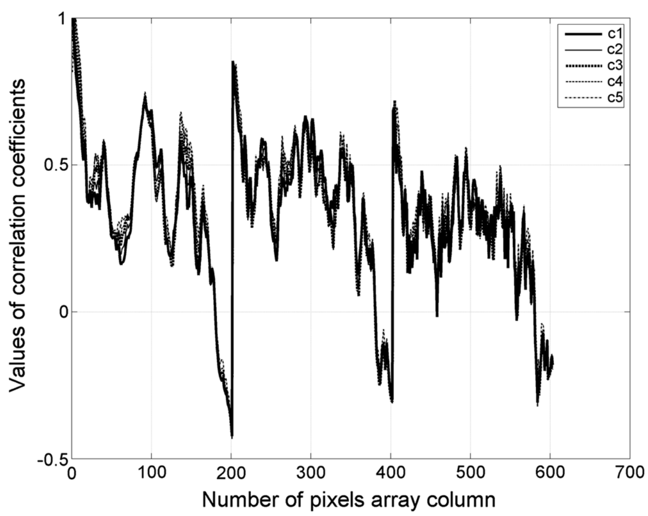

Fig. 3 The changes of the correlation coefficients between the five columns of pixel arrays of the digital image of the OS retina.

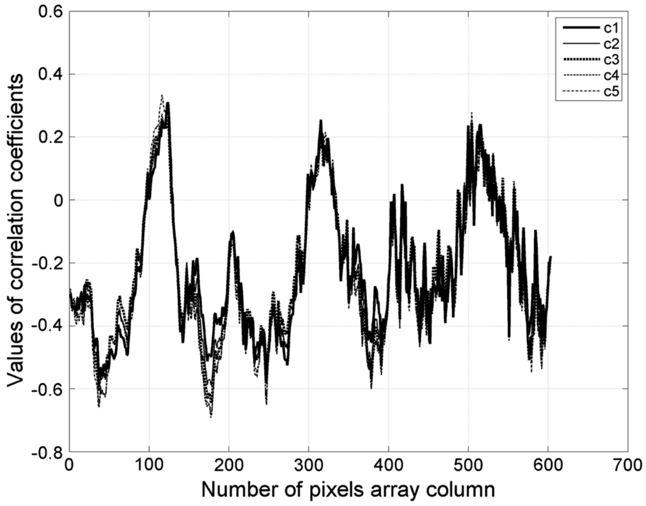

Fig. 4 The changes of the correlation coefficients between the five columns of pixel arrays of the digital images of the OD and OS retinas.

$k = 13000$. A normalized autocovariance function of the digital image of the retina OS is presented in Fig. 6. The values of the correlation coefficient are changing from $r = 1.0$ at $k = 0$ to $r < 0.1$ when $k = 10000$. Last results show the decreasing of the correlation coefficient values to $r \rightarrow 0$ at a large value of $k = 10000$.

A normalized covariance function of the digital images of the OD and OS retinas is shown in Fig. 7.

The values of covariance function at $k = 0$ are not big $(-0.25 < r < 0.25)$; however, when quantise interval is increasing, the values of covariance function are slightly increasing too, and when $k = 20{,}000$ they are changing in the interval $-0.3 < r < 0.45$.

A scatter in percentages of the correlation coefficient matrix values of the digital images of the OD and OS retinas is shown in Fig. 8.

The negative correlation occupies about 50% of both array areas. Graphical view of the spatial matrix of the correlation coefficients of the digital images of the OD and OS retinas is presented in Fig. 9.

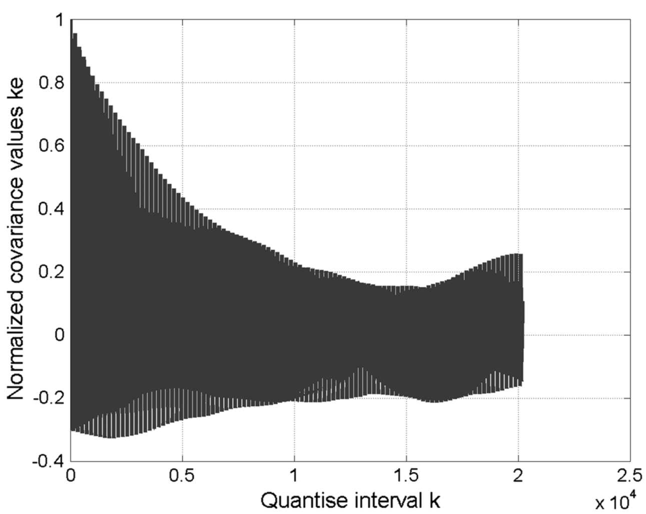

Fig. 5 A normalized autocovariance function of the digital image of the OD retina.

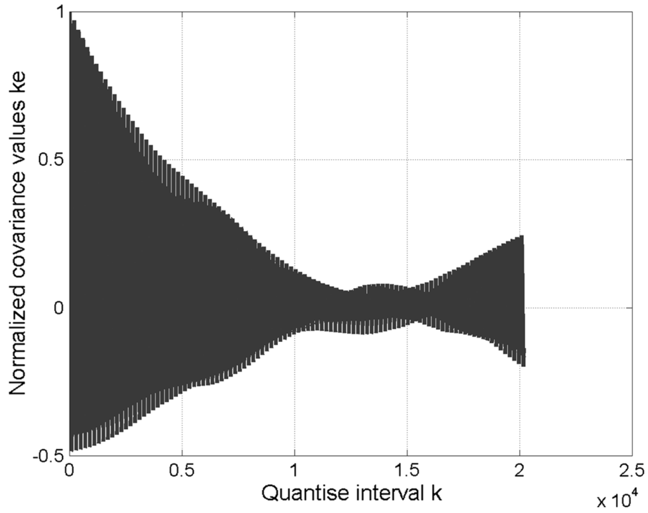

Fig. 6 A normalized autocovariance function of the digital image of the OS retina.

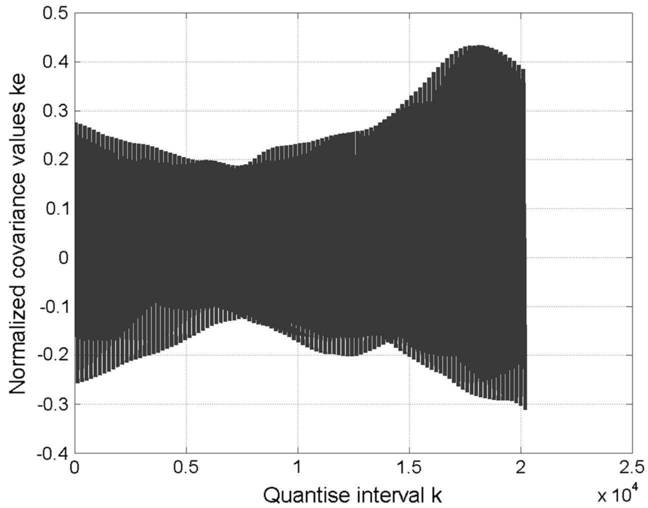

Fig. 7 A normalized covariance function of the digital images of the OD and OS retinas.

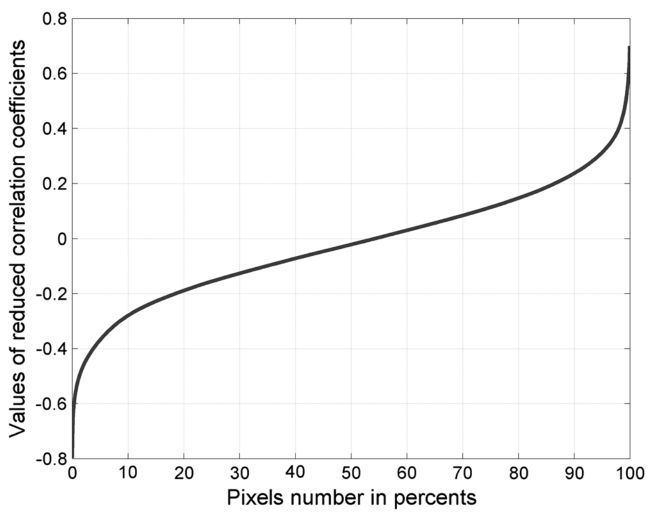

Fig. 8 The changes in percents of the matrix of the covariance coefficients of OD and OS retinas.

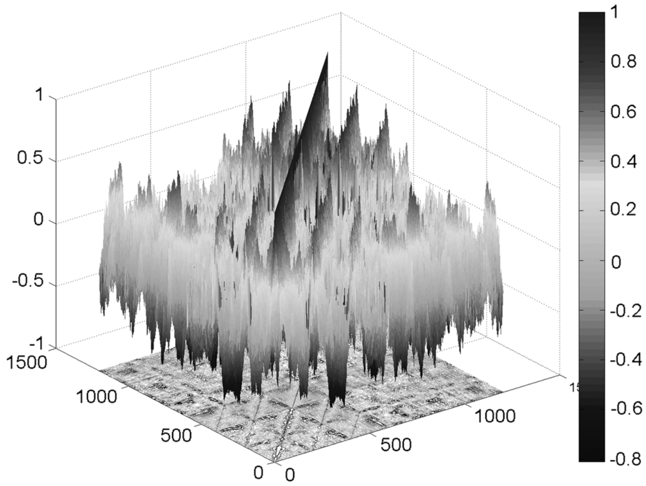

Fig. 9 Graphical view of the spatial matrix of the correlation coefficients of the digital images of the OD and OS retinas.

5 Conclusions

1. The mean correlation between the pixel columns of the arrays of the digital images of the human eye OD and OS retinas along the single RGB component is not big or even negative. However, the values of the correlation coefficients between pixel columns of the OD and OS arrays applying total RGB color tensors are near one. This means that OD and OS retina correlation is very strong.
2. A normalized autocovariance function of the digital image allows us to determine the change of the correlation depending on the pixel quantise interval. The estimations of the normalized covariance functions do not differ much when applying the different RGB spectrum components. The values of a normalized covariance function of the digital images of the OD and OS retinas are decreasing very slowly and approach zero $r \to 0$ at a large value of the quantise interval $k = 10000$. This shows a very big correlation between the pixels, which are close to each other. The values of a normalized covariance function of the digital images of the OD and OS retinas are not big ($-0.2 < r < 0.45$) in all the quantise intervals. This shows that correlation between OD and OS retinas is weak.
3. It was detected that negative correlation between the pixels occupies ∼50% of both OD and OS retina array areas and that shows the graphical view of the spatial matrix of the correlation coefficients.

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