

# Strategy of teaching students the design of optical systems in the interests of the optical industry

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## ABSTRACT

The difficulty in designing new optical systems lies in the fact that design remains largely an “art” that depends on the experience and intuition of the developer, rather than a precise engineering discipline that strictly and unambiguously algorithms the design process. This circumstance is quite noticeable in the optical industry, where young specialists need time to adapt to practical work on the design of optical systems. These circumstances lead to the need to pay special attention to teaching programs of optics education that will ensure that students receive the necessary competencies for the successful fulfillment of production tasks for the development of complex optical systems in industry. The report will present our proposals on the necessary refinement of master's programs of study and the formation of joint training programs for lens designer.

**Keywords:** Lens design, synthesis, optimization, image quality, classical aberrations, orthogonal aberrations, teaching programs

## 1. INTRODUCTION

The technical level of the developed optoelectronic products is primarily determined by the quality of the optical systems used, the optical parameters of which determine the tactical and technical characteristics of the products. Requirements for designed optical systems often approach the maximum achievable in terms of image quality at increased values of the lens aperture and field of view and when using the minimum possible number of optical elements and reduced weight and size characteristics.

This circumstance determines the important role of designer of optical systems in the process of creating new optoelectronic products, their professional training, knowledge of the basic laws of optical image formation and, most importantly, understanding how to use this knowledge to create new optical systems that meet the requirements of the technical specifications for the development.

In a recent report by V. Mahajan "Roadmap for teaching optical imaging and aberrations" [1], overview of the content of training courses on Gaussian imaging, Paraxial ray tracing, geometrical optics, optical aberrations, diffraction theory that provide basic theoretical knowledge for optic systems designer. However, there is some gap between basic knowledge of optics for lens designer (computational optics) and practical work on the design of new optical systems. So far, the design of optical systems is more an art than a science. Many design procedures are based on the experience and intuition of the designer, which has been formed in optical system designers over many years of intense creative work.

Let's analyze the main reasons for this state of affairs.

## 2. MAIN PROBLEMS OF THE PROCESS OF OPTICAL SYSTEM DESIGN

Consider the main stages of the optical system design:

### 1. Agreement of specification requirements.

This is completely heuristic stage. It is very impotent but in the literature there are no constructive recommendations for its implementation.

### 2. Design of the starting optical system (synthesis).

This is in general heuristic stage. This stage is missing in existing software. On this stage lens designer only can choose the starting optical system from previously developed systems or from patent information. Note that in the pre-computer era, manual methods for constructing optical systems were widely developed and used but now modern software cannot create a starting optical system according to the requirements of specification.

### 3. Analysis.

Analysis of optical systems is fully software supported. However, the appropriate methods for analyzing the correction capabilities of specific types of optical systems and analyzing their limiting characteristics have not been sufficiently developed.

#### 4. Optimization;

The existing optimization software's have significant local limitations due to the multi-extremity of the minimized merit function and, in most cases, optimization does not lead to an acceptable solution, but only to finding the nearest local extremum. This drastically reduces the efficiency of using optimization programs.

The current state of optics design can be summed up by the following excerpt from the Zemax Tutorial of one of the most widely used programs and, we note that it is very good software, which we use in the educational process. This excerpt note: "What does ZEMAX not do? Neither the ZEMAX program nor the ZEMAX documentation will teach you how to design optical systems. Although the program will help you in the design and analysis of optical systems, you are still the main designer. The ZEMAX documentation is not a tutorial on optical design, terminology, or methodology."

In this regard, in the training programs for specialists in the design of optical systems for the master's level, we paid special attention to the following training courses:

- development of the classical theory of third-order aberrations and its practical application for constructing starting (initial) optical systems based on the requirements of the specification (synthesis of optical systems);
- semi-heuristic construction of initial optical system configurations;
- theory of orthogonal aberrations and its use in the development of methods for studying aberration properties and limiting characteristics of optical systems;
- carrying out by students independent practical work on the study of aberration properties and limiting characteristics of various types of optical systems contained in databases and so on.

Let us consider in more detail some directions of the taught courses of the master's program of the specialization in the design of optical systems. Our goal is to teach students the practical design of optical systems and with them to join step by step the science of optical design with the art of optical design.

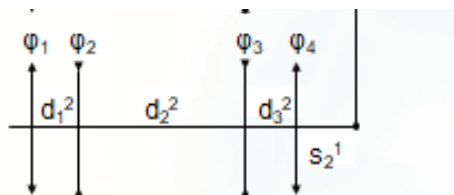
### 3. SYNTHESIS OF OPTICAL SYSTEMS

A detailed analysis of possible directions for the development of the theory and methods of design the initial optical system according to the requirements of the specification showed that it is most expedient and effective to use the analytical methods of paraxial optics and the theory of third-order aberrations developed in the pre-computer era [2-3]. At the same time, analytical methods should be supplemented with semi-heuristic criteria for the rationality of the structure of optical systems depending on the required external characteristics (aperture, field of view, etc.)

This approach made it possible to formulate a technique (synthesis methodology) for synthesis of the initial optical system, consisting of 4 interconnected stages [4-5].

#### Stage 1.

At the first stage, a thin-component model of the optical system (layout) is constructed, which consists of a number of simpler subsystems (thin components) and distances between them.



At this stage, the number of components, their design parameters (optical powers of individual components  $\phi_i$ , distances between them  $d_i$ , position of the aperture diaphragm, etc.) are variable and are determined from the conditions for fulfilling a number of requirements of the specification and fulfilling semi-heuristic criteria for the rationality of the layout.

The semi-heuristic criteria include, among others:

- the conditions for the optimal distribution of the field of view and aperture (speed) by components;
- the angular magnification in the pupils of the entire system should not go beyond some limits (generalization of Berek's criterion [4]) that depend on the field of view and speed and other criteria and so on.

### Stage 2.

At the second stage, the requirements for the aberration correction of individual components are determined.

To assess the joint effect of third-order aberrations on image quality, we have formed an integral characteristic of image quality - mean square of geometric aberrations  $M$  over field of view-aperture region, which allows us to evaluate the image quality of an optical system with a one number

$$M = \frac{2}{\pi} \int_0^1 \int_0^{2\pi} \int_0^1 (\delta g'^2 + \delta G'^2) r dr \rho d\rho d\varphi - 2 \int_0^1 \left( \frac{1}{\pi} \int_0^{2\pi} \int_0^1 \delta g' \rho d\rho d\varphi \right)^2 r dr \quad (1)$$

We introduce an orthogonal expansion of geometric aberrations and associate the coefficients of the orthogonal expansion with the Seidel sums of each thin component. Then, for the integral characteristic of the image quality, we obtain the final expression in quadratic form through the Seidel sums of each component, which are unknown and we determine them at this stage. Note that very convenient expressions for third-order aberrations for optical systems consisting of thin components were obtained by G.G. Slyusarev [4].

To limit the number of possible solutions and select the most expedient of them, it is very important to use a number of additional semi-heuristic conditions, which ensure both the construction of optical systems with realizable configurations of individual components and at the same time obtaining of optical systems with small higher-order aberrations [4-5].

On this stage we reduced the problem of designing a complex optical system to simpler subtasks of designing thin individual components.

### Stage 3.

Selection of configurations and design of individual components.

According to the data obtained in stage 2 (required aberration correction of each component), the software determines the number of lenses of each component, the radii of curvature, the required types of optical glass for each lens.

### Stage 4.

Assembling the system from the components built in stage 3. At this stage the software introduces thicknesses in each thin component (the thicknesses of individual lenses and the air gaps between them), unites components into a single optical system and we get initial or starting optical system.

## 4. DEVELOPMENT OF THE THEORY OF ORTHOGONAL ABERRATIONS

As is known, in the case of axisymmetric (rotationally symmetric) systems, the wave aberration depends on three rotation invariants

$$r^2, \rho^2, r\rho\cos\varphi. \quad (2)$$

Usually, the wave aberration is represented as an expansion in a power series and the coefficients of this expansion are called the aberration coefficients of individual aberrations. The classical expansion of the wave aberration function has the form

$$V = \sum_{lkm} b_{lkm} r^{m+2l} \rho^{m+2k} \cos^m \varphi \quad (3)$$

where  $b_{lkm}$  are aberration coefficients.

In this case, the individual terms of expansion (3) are the classic individual aberrations

$$b_{lkm} r^{m+2l} \rho^{m+2k} \cos^m \varphi \quad (4)$$

and the order  $N$  of an individual aberration is determined by the total degree of the variables  $r$  and  $\rho$  included in its expression, reduced by one

$$N = 2(m+1+k) - 1.$$

If we restrict the degree of variables to the 3rd degree, then these are third-order aberrations. Aberrations determined by the degree of variables 5 and higher are called higher-order aberrations.

### 4.1. Determination of orthogonal aberrations.

If we form in a special way a complete system of orthogonal polynomials in a three-dimensional domain of definition of wave aberration (we will call this domain for brevity the field of view-aperture region)  $0 \leq r \leq 1, 0 \leq \rho \leq 1, 0 \leq \varphi \leq 2\pi$ , we

can represent the wave aberration in the form of an expansion in terms of an orthogonal system of functions [5-7]. As will be shown below, such a representation of wave aberration makes it possible to define a new class of individual aberrations - orthogonal individual aberrations. These aberrations have a number of unique properties and allow the development of fundamentally new approaches in the aberration analysis of optical systems, the determination of their limiting correction capabilities, the creation of new effective approaches to the design of optical systems, largely taking into account the physical laws of their functioning.

It is most simple to form such a system of polynomials that take into account the structure of the dependence of wave aberration on the variables  $r$ ,  $\rho$ , and  $\varphi$  in the domain of definition of wave aberration. This is most easily solved by generalizing the Zernike polynomials and introducing into their structure a polynomial that depends on the field coordinate  $r$  and takes into account the structure of the dependence of the wave aberration on the field coordinate (2). As such a polynomial, we used a radial polynomial  $R_{m+2l}^m(r)$ , where the index  $l$  varies from 0 to a certain value  $L$ . As a result, the complete orthogonal system of polynomials in the field of view-aperture region  $0 \leq r \leq 1$ ,  $0 \leq \rho \leq 1$ ,  $0 \leq \varphi \leq 2\pi$  is defined as follows

$$\begin{aligned} Z_{lkm}(r, \rho, \varphi) &= \varepsilon_m R_{m+2l}^m(r) R_{m+2k}^m(\rho) \cos m\varphi \\ \bar{Z}_{lkm}(r, \rho, \varphi) &= \varepsilon_m R_{m+2l}^m(r) R_{m+2k}^m(\rho) \sin m\varphi \end{aligned} \quad (5)$$

Obviously, the new system of polynomials (5) is an orthogonal system of polynomials in the three-dimensional region  $0 \leq r \leq 1$ ,  $0 \leq \rho \leq 1$ ,  $0 \leq \varphi \leq 2\pi$ , where the orthogonality in the field variable  $r$  is provided by the radial polynomial  $R_{m+2l}^m(r)$ , the orthogonality in the variable  $\rho$  - by the radial polynomial  $R_{m+2k}^m(\rho)$ , and the orthogonality in the variable  $\varphi$  - by the orthogonality of the trigonometric functions.

Orthogonality of polynomials (5) is expressed as follows

$$\int_0^{2\pi} \int_0^1 \int_0^1 Z_{lkm} Z_{l'k'm'} r dr \rho d\rho d\varphi = U_{lkm} \delta_{ll'} \delta_{kk'} \delta_{mm'}$$

where  $\delta_{ll'}$  is the Kronecker-Capelli symbol,

$U_{lkm} = \pi / (4(m+2k+1)(m+2l+1))$  is the normalization constant.

In this case, the wave aberration can be represented as an orthogonal expansion [5-7]

$$\begin{aligned} V &= \sum_l \sum_k \sum_m A_{lkm} Z_{lkm}(r, \rho, \varphi) = \\ &= \sum_l \sum_k \sum_m A_{lkm} \varepsilon_m R_{m+2l}^m(r) R_{m+2k}^m(\rho) \cos m\varphi \end{aligned} \quad (6)$$

where each term in the expansion is a separate orthogonal aberration

$$V = A_{lkm} \varepsilon_m R_{m+2l}^m(r) R_{m+2k}^m(\rho) \cos m\varphi \quad (7)$$

It is necessary to immediately emphasize the most important structure of individual orthogonal aberrations, namely: by its nature, any individual orthogonal aberration (7) consists of a well-defined finite sum of classical individual aberrations (4). As can be shown, the individual classical aberrations included in the orthogonal separate aberration are optimally balanced in their structure with each other to obtain the maximum value of the mean square of the wave aberration in the considered field of view-aperture region.

To use orthogonal aberrations in the practice of optical system design, it is necessary to develop a method for determining the expansion coefficients  $A_{lkm}$  expression (6). The most natural and effectively is the direct use of the orthogonality property of the polynomials which leads to the following expression

$$A_{lkm} = \frac{1}{U_{lkm}} \int_0^{2\pi} \int_0^1 \int_0^1 V(r, \rho, \varphi) Z_{lkm} r dr \rho d\rho d\varphi \quad (8)$$

Sometimes, for a more visual assessment of the results obtained in practical work, it is convenient to use an orthonormal system of polynomials, which is related to orthogonal polynomials (5) as follows

$$B_{lkm}(r, \rho, \varphi) = \frac{1}{2\sqrt{m+2l+1}\sqrt{m+2k+1}} Z_{lkm}(r, \rho, \varphi) \quad (9)$$

$$\bar{B}_{lkm}(r, \rho, \varphi) = \frac{1}{2\sqrt{m+2l+1}\sqrt{m+2k+1}} \bar{Z}_{lkm}(r, \rho, \varphi)$$

The expansion of the wave aberration (9) in terms of this orthonormal system of polynomials can be written in the following form

$$V = \sum_l \sum_k \sum_m a_{lkm} B_{lkm}(r, \rho, \varphi) \quad (10)$$

and the coefficients  $a_{lkm}$  and  $A_{lkm}$  are related by the following relation

$$a_{lkm} = \frac{A_{lkm}}{2\sqrt{m+2l+1}\sqrt{m+2k+1}}$$

The developed approach describing the aberration properties of optical systems by means of orthogonal aberrations and its use in the construction of new methods and techniques to the design of optical systems form a new section of theory of lens design, namely, "The theory of orthogonal aberrations and its applications in the design of optical systems."

## 4.2. The Definition of the Integral characteristic of image quality

For an integral evaluation of the image quality of an optical system, it is convenient to introduce the concept/notion of the mean square of the wavefront aberration  $F$  in the field of view-aperture region (we will call this quality characteristic the integral aberration functional) [5-8]

$$F = \frac{1}{U} \frac{1}{S} \iint_S \iint_U ((V - \bar{V})^2) dS dU \quad (11)$$

where  $S$  is the domain of integration determined by the set of all rays passed through the optical system for the given point of the object, also denotes the area of the domain  $S$ ,

$U$  is the region of integration, defined by the region of the angular field of view (also denoting the area of this region),

$\bar{V}$  is the average value of the wave aberration in this region.

When using the orthogonal representation of wave aberrations, the integral aberration functional in the field of view-aperture region takes on a simple and very important form

$$F = \frac{1}{4} \sum_l \sum_k \sum_m \frac{A_{lkm}^2}{(m+2l+1)(m+2k+1)} \quad (12)$$

moreover,  $m$  and  $k$  are not equal to zero at the same time.

Relation (12) shows that mean square of the wave aberration (the integral aberration functional  $F$ ) is very simply expressed through the coefficients  $A_{lkm}$  of the expansion (6) of the wave aberration, and each individual orthogonal aberration decreases the functional  $F$  independently of each other. Moreover, further we will show a number of other very important properties of the orthogonal representation (6) of wave aberration.

Usually aberrations of orders 1 and 3 are called aberrations of lower orders, and aberrations of orders 5, 7 and higher are called aberrations of higher orders. Let us denote the contribution of lower-order aberrations to the deterioration of the image quality through  $F_l$ , and the contribution of higher-order aberrations - through  $F_h$ . Then

$$F = F_l + F_h \quad (13)$$

It is useful sometimes to consider the values of  $F_l$ ,  $F_h$ , in percentage terms relative to  $F$ . We will denote them through  $F_l\%$ ,  $F_h\%$

$$F_l\% = (F_l \setminus F) * 100\%, \quad F_h\% = (F_h \setminus F) * 100\%.$$

These values show the percentage contribution of lower and higher order aberrations to image deterioration.

## 5. ORTHOGONAL ABERRATIONS AND SOME RESULT OF STUDY OF THE ABERRATION PROPERTIES OF OPTICAL SYSTEMS

Practical work using orthogonal aberrations made it possible to reveal a number of new aberration properties of optical systems [5-8].

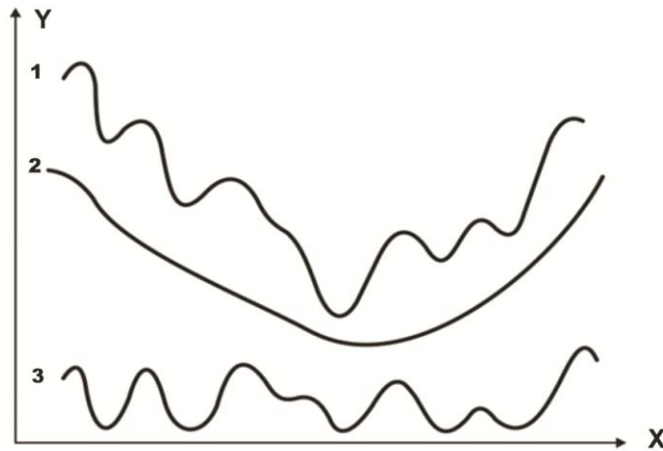
1. If the wave aberration of an optical system contains only higher-order orthogonal aberrations, this means that classical higher-order aberrations are completely balanced by lower-order aberrations and the desire to improve the system by introducing controlled lower-order aberrations will only worsen it.

2. When changing the design parameters of the optical system, including optimization, higher-order aberrations (fifth and higher orders) change to a small extent compared to the change in lower-order aberrations (the property of invariance of higher-order aberrations). Table 1 shows the values of the aberration functionals F, F<sub>1</sub>, F<sub>n</sub> for a number of optical systems before and after local optimization using the ZEMAX program.

**Table 1.** Values of aberration functionals F, F<sub>1</sub>, F<sub>n</sub> of a number of optical systems before and after local optimization. Note: In each line the considered characteristics are given before and after optimization .

№	Patent	Number of components / lenses	F	F <sub>1</sub>	F <sub>n</sub>	F <sub>n</sub> %	Polynomial degrees in r, ρ, φ	Order of aberrations
1	59-8803-1	5 \ 9	0.897	0.575	0.322	35.9	5,6,4	7
			0.304	0.068	0.236	77.6	5,6,4	7
2	59-8803-2	5 \ 9	0.839	0.531	0.258	30.8	5,6,4	7
			0.293	0.080	0.213	72.6	5,6,4	7
4	3833946	7 \ 14	1.747	1.674	0.073	4.2	4,6,3	5
			0.116	0.040	0.076	65.4	4,6,4	7
5	59-8803-3	5 \ 9	1.734	1.252	0.482	27.8	5,6,4	7
			0.556	0.176	0.380	68.3	5,6,4	7
7	1256435-1	6 \ 11	2.221	1.994	0.227	10.2	5,6,2	7
			0.345	0.145	0.200	58.1	5,6,3	7
8	1256435-2	6 \ 11	3.927	3.593	0.330	8.4	4,6,2	7
			0.639	0.342	0.297	46.5	5,6,4	7
9	4260223	8 \ 14	6.730	6.010	0.720	10.7	5,5,3	5
			0.689	0.252	0.437	63.4	5,7,3	7
15	62-143011	8 \ 14	19.589	18.923	0.666	3.4	4,4,3	5
			0.652	0.310	0.342	52.5	6,6,3	7
20	3359057	3 \ 6	172.35	166.84	5.515	3.2	2,6,2	5
			31.37	26.28	5.145	16.4	5,6,3	5
24	4671626	5 \ 10	1015.8	1015.1	0.68	0.067	2,4,2	3
			1.0	0.477	0.531	52.7	5,6,2	5

3. The rapid change in lower-order aberrations in comparison with higher-order aberrations proves that lower-order aberrations are responsible for the multi-extremity of the minimized merit function in optimization programs, and higher-order aberrations determine the slowly changing part of the merit function (Fig. 1). So this confirms the fact that, in local optimization, the minimization of the aberration functional is carried out mainly by balancing higher-order aberrations with lower-order aberrations and reducing the contribution of lower-order aberrations



**Figure 1.** 1 – merit function F

2 – merit function defined by higher order aberrations F<sub>h</sub>

3 – merit function defined by lower order aberrations F<sub>l</sub>

Using the contribution of higher-order aberrations F<sub>h</sub> as the aberration part of the merit function in optimization programs we can significantly reduce the number of local extrema of the merit function by eliminating from the merit function rapidly changing lower-order aberrations that determine the multi-extremity of the merit function. This significantly reduces the local limitation of optimization programs and allowed us to create pseudo-global optimization programs [5-7]. Our experience has shown that pseudo-global optimization programs make it possible to obtain optical systems with reduced higher-order aberrations and to achieve the highest possible image quality of the optical system being optimized.

4. In the local minimum, the contribution of higher-order aberrations (aberration balancing) is usually from 60 to 70 percent, that is

$$F_h\% = (60 - 70)\%.$$

5. For most of the optical systems, the design parameters of which were obtained from patent information or databases (LensView, Zabase) the contribution of higher-order aberrations is less than 40%. This means that the design parameters are deliberately distorted by the developer when published, and the aberration characteristics of the optical systems, calculated according to the indicated design parameters, do not allow correct comparison of the considered systems.

6. Higher-order aberrations make it possible, even with incorrectly specified parameters, to conduct a more reliable comparison of the aberration characteristics of optical schemes with each other without optimization, to evaluate the possible aberration correction of the system after optimization, and to make a decision on the prospects of using the analyzed optical systems.

## **6. ANALYSIS OF THE LIMITING ABERRATION CHARACTERISTICS OF OPTICAL SYSTEMS**

The most important external characteristics of optical systems, which determine their region of application, the complexity of their development, as well as aberration properties, are aperture and field of view. It is these characteristics that determine the domain of the field of view-aperture region, in which the aberration properties of optical systems are determined.

The aperture and field of view determine a unified region of space, in which the object space is transformed into the image space, and the complexity of the implementation of such a transformation, as well as the complexity of the developed optical system and its configuration. It is most natural and useful to combine these two characteristics into a single integral characteristic. As such a characteristic, it is most natural to determine the volume of the field of view-aperture region. The volume of the field of view-aperture region determines the complexity of the implementation of the requirements of the technical specifications for the development of an optical system and, naturally, the complexity of the resulting optical system and its aberration properties.

Let us define the volume  $\Omega$  of the four-dimensional the field of view - aperture region, in which the aberration functional are calculated as follows

$$\Omega = \iint\limits_{US} dSdU , \tag{14}$$

where the area  $S$  - the area of integration, determined by the set of all rays passing through the pupil  $S$  of the optical system, denotes at the same time the area of the area  $S$ ;

$U$  - the region of integration, determined by the set of all points of the image, simultaneously determines the area of the region  $U$ .

### 6.1. LIMITING VALUES OF THE FIELD OF VIEW AND APERTUE OF THE ANALYZED OPTICAL SYSTEMS

By changing the aperture (region  $S$ ) or the field of view (region  $U$ ), we also change the volume of the field of view-aperture region. However, it is possible to simultaneously change the aperture and the field of view so that the volume of the field of view-aperture region remains constant, i.e.

$$\Omega = \text{const} .$$

Naturally, depending on the type of optical system, it is convenient to represent the expression for the volume of the angular field-aperture region  $\Omega$  in one form or another. Consider lens-type optical systems (object at infinity, image at a finite distance). For the indicated type of optical systems, we use the following expression to determine the volume  $\Omega$

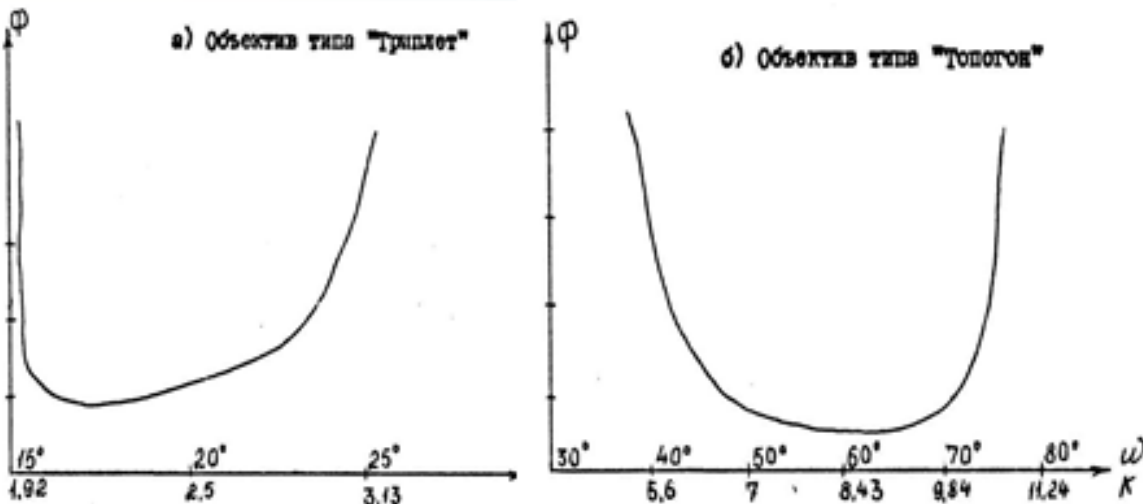
$$\Omega \sim \omega^2 \sigma'^2 \sim \frac{\omega^2}{K^2} , \tag{15}$$

where  $\omega$  is half of the angular field of view ,

$\sigma'$  - back aperture angle,

$K$  - the f-number.

Our studies have shown [8] that within a fairly wide range of changes in the angular field (region  $U$ ) and aperture (region  $S$ ) while maintaining the constancy of the volume of the field of view-aperture  $\Omega = \text{const}$ , the aberration functional  $F$  (or  $F_h$ ) of the optical system under study remains almost constant, that is, it has a very flat minimum (Figure 2.)



**Figure 2,** Changes in the aberration functional  $F$  upon a special change in the angular field view and f-number

The region of relatively slow change in the aberration functional (let's call it the admissible region) is limited to the right and to the left by the regions of a sharp increase in the functional  $F$ , which determine the limiting values of the angular field of view and the aperture  $\omega_{lim}, \sigma'_{lim}$ , that is, the limiting capabilities of optical system. The indicated property of the behavior of the aberration functional is fulfilled for all optical systems. Consequently, each optical system has the

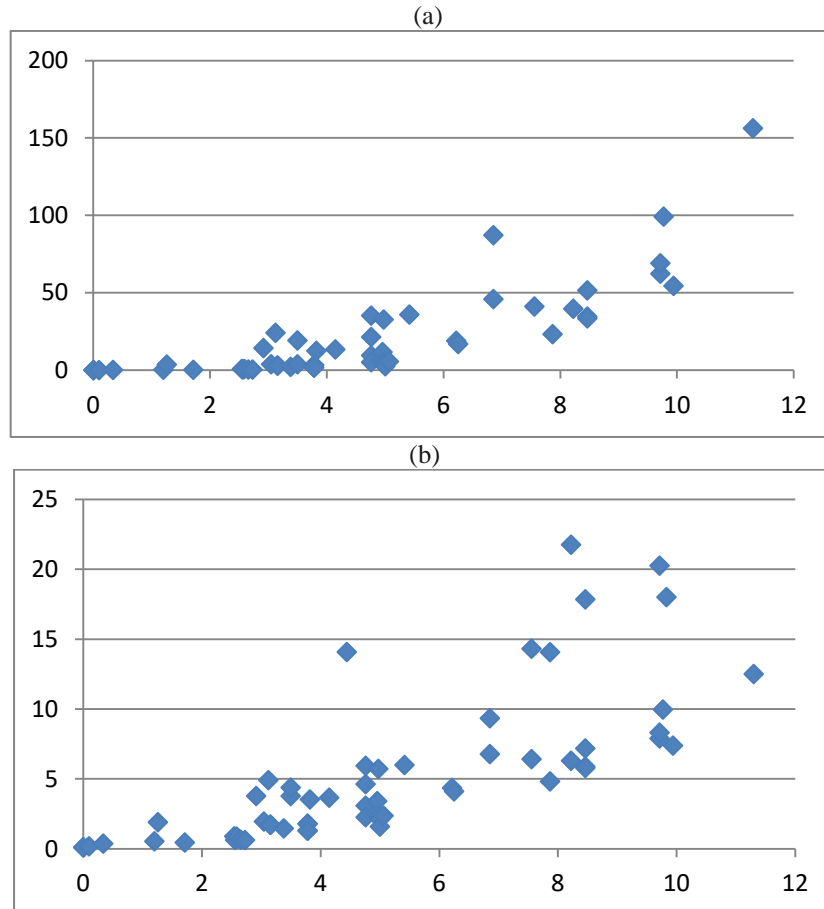


limiting values of the angular field of view and aperture  $\omega_{lim}, \sigma'_{lim}$  (Klim), which can be determined based on the above analysis of the areas of sharp increase in the aberration functional.

The specified limit values determine the limiting capabilities of specific optical systems and make it possible to choose the most optimal design solutions for the given values of the angular field of view and aperture in the design specification.

## 6.2. MAXIMUM POSSIBLE ABERRATION CORRECTION OF OPTICAL SYSTEMS OF DIFFERENT COMPLEXITY

The study of the maximum achievable image quality of various types of optical systems (for example triplet, Gauss lens and so on) for different fields of view and apertures of their application can be carried out using a special study of the behavior of the aberration functionals. Let us consider the behavior of the aberration functional of a specific type of optical systems depending on the volume of the field of view-pupil region of their application. For each type of systems, there are minimum attainable values of aberration functionals, depending on the size of the volume of the field of view-aperture region (Diagram 1). These minimum values of the aberration functionals determine the maximum possible aberration correction of the considered type of optical systems and they can be analytically approximated by some simple function (in the first approximation, the quadratic dependence of F on  $\Omega$ ). To calculate the volume of the angular field of view-aperture region we use formula (15), express half of the angular field of view in radians.



**Diagram 1.**

The aberration functionals F (a) and  $\sqrt{F}$  (b) on the volume of the field of view-aperture region for Gaussian lenses.

As a practical example, let us show the dependence of the values of the aberration functional F and  $\sqrt{F}$  on the volume of the field of view-aperture region for Gaussian lenses. Specific lens design parameters were obtained from the ZEBASE (Section L) and LensView databases (all lenses have a focal length of 100 mm). It should be noted right away that in the

databases the design parameters of the lenses are to some extent deliberately damaged by the developer. Therefore, sometimes we get a large spread in the aberration characteristics of lenses. The results are shown in the diagram. The abscissas is the volume  $\Omega$  of the field of view-aperture region, and the ordinate is aberration functions  $F$  and  $\sqrt{F}$ . The results clearly show the existence of a limiting aberration correction for specific optical types of schemes. These studies have shown that with the help of special processing of the data contained in the database of ready-made optical systems, it is possible to extract information encoded in it about the maximum attainable aberration characteristics of optical systems of varying complexity and types, the relationship between structural and aberration characteristics, etc. This gives possibility to create intelligent knowledge bases that provides further formalization of the heuristic stages of designing optical systems. We began with our students to carry out the elements of this work in practical classes and to study the aberration properties of various types of optical systems and their limiting characteristics.

### 6.3. GENERALIZED INDICATOR OF THE COMPLEXITY OF THE OPTICAL SYSTEMS

In the previous section, we studied the behavior of aberration functionals for individual optical systems with the same focal length. However, when analyzing the achievable aberration correction of various optical systems and comparing the analysis results with each other, it is necessary to take into account that the value of both geometric and wave aberrations with a change in the focal length changes in proportion to the change in the focal length. Therefore, to assess the complexity of the requirements of specifications (and simultaneously designed optical system) it is useful to introduce a generalized indicator of complexity  $D$  of the optical systems (and requirements of the specification). The introduced indicator of the complexity of the optical system is defined as the product of the volume of the field of view-aperture region  $\Omega$  by the focal length  $f'$  of the lens, that is

$$D = \Omega \times f' \tag{16}$$

Hence, in the case of lens-type optical systems, the complexity indicator can be written as

$$D = \omega^2 \sigma^2 f' = \frac{\omega^2}{K^2} f' \tag{17}$$

Our studies have shown the important role of using this indicator in the practical work of lens designer.

## 7. EXAMPLES OF PRACTICAL USE OF THE ORTHOGONAL ABERRATIONS IN LENS DESIGN

### 7.1. EXAMPLE 1

Let us show a concrete example of the use of orthogonal aberrations in the analysis of the correctional possibilities of optical systems. Suppose that we are engaged in working out an objective with the parameters:  $F/\# = 2$ , and  $2\omega = 45$  degree. The search for possible prototypes in a database has given us two prototypes approximately identical in characteristics from the patent USA No. 4.123.144 (variants No. 3 and No. 7). Both prototypes have approximately identical image quality. There is a question regarding which of the prototypes has the best correction possibility. The standard analysis of aberrational characteristics does not give the straightforward answer to this question. Let us estimate the contribution of aberrations of the higher-order  $F_h$  to aberration functional  $F$  for each of the objectives (Table 2).

**Table 2.** Aberrational characteristics of lenses (Patent US4123144; Examples 3 and 7 of 9).

No	Aberration functional			Balancing of aberrations	
	F	F <sub>I</sub>	F <sub>h</sub>	F <sub>I</sub> %	F <sub>h</sub> %
3	15.63	8.61	7.02	55%	45%
7	15.29	11.72	3.57	77%	23%

The aberration correction of the two objectives is almost equal, but the analysis of the higher-order aberrations shows that the contribution of aberrations of the higher-orders in scheme No. 7 is much lower (twice) than that in scheme No. 3. It is possible to expect that the potential correctional possibilities of objective No. 7 are better. Moreover, it is even possible to predict approximately (to estimate before optimization) the possible level of aberrational correction of objectives after optimization. Thus, with some approach, it is possible to consider that, after optimization, the higher-

order aberrations will decrease but not more than twice, and contributions of the low-order and high-order aberrations to aberration functional (balancing of aberrations) will be approximately 60%. These assumptions allow giving the values shown in Table 3 for aberration functionals after optimization.

**Table 3.** Predicted values of aberrational characteristics of lenses after optimization (Patent US 4123144; Examples 3 and 7 of 9)

No	Aberration functional			Balancing of aberrations	
	F	Fl	Fh	Fl%	Fh%
3	5.83	2.33	3.50	40%	60%
7	2.92	1.17	1.75	40%	60%

The results of the optimization of analyzed schemes with the use of Zemax are shown in Table 7. As a result of optimization the aberration functional F in the more perspective scheme 7 has decreased by 5.75 times, while that in the less perspective scheme F has decreased only by 2.84 times. Thus, as one would expect, the contribution of the higher-order aberrations decreases much less than that of the low-order aberrations. It is important to note good results of the forecast of results of optimization (Tables 3 and 4).

**Table 4.** Aberrational characteristics of lenses (Patent US4123144; Examples 3 and 7 of 9) after optimization

No	Aberration functional			Balancing of aberrations	
	F	Fl	Fh	Fl%	Fh%
3	5.50	1.56	3.94	28%	72%
7	2.66	0.98	1.68	37%	63%

## 7.2. EXAMPLE

Consider the Gaussian lenses listed in the ZEBAZE Database, Section L. Analysis of the aberration functional of these lenses depending on the volume of the angular field of view-aperture region shows that there is a large scatter of F values even for schemes with approximately the same apertures and angular fields of view, that is, used at the same values of the volume of the field of view-aperture. Let's choose a lens L\_012 for which the aberration correction is much worse than the maximum possible correction. For this lens  $f^* = 100$ ,  $2\omega = 43$  degree,  $F/\# = 1.2$ ,  $\Omega = 9.83$ . Table 5 shows its aberration characteristics. As we can see from diagram 1, for  $\Omega = 9.83$  its aberration characteristics should be much better, maybe approximately  $F = 50 - 60$ . Also, as we can see, optimal balancing of higher-order aberrations with lower-order aberrations has not been achieved. Let's optimize this lens.

**Table 5.** Aberrational characteristics of lens L\_012

L_012	Aberration functional			Balancing of aberrations	
	F	Fl	Fh	Fl%	Fh%
	325.4	214.7	110.57	66%	34%

First we use the software complex ZEMAX. As a result of the optimization, the following results were obtained (Table 6). As we expected, we obtained the mean square of the wave aberration over the field of view-aperture region ( $F = 58.1$ ) close to the predicted value ( $F = 50 - 60$ ). However, the resulting balancing of aberrations (67% - 33%) does not match the balancing that should be obtained with a good design

**Table 6.** Aberrational characteristics of lens L\_012 after ZEMAX optimization.

L_012 Opt.	Aberration functional			Balancing of aberrations	
	F	Fl	Fh	Fl%	Fh%
	58.1	38.9	19.2	67%	33%

We now use an optimization program that uses orthogonal aberrations and separates the contributions of aberrations of different orders in the merit function. After several optimization modes, a very interesting solution was obtained (Table 7). We got a noticeable improvement in aberration correction, but the most interesting thing is different. In the resulting optical system, almost perfect balancing of higher-order aberrations with lower-order aberrations, which is quite rare. The contribution of harmful first and third order aberrations to image degradation is approximately 4%. Higher order aberrations contribute 96% to the mean square of the wave aberration.

**Table 7.** Aberrational characteristics of lens L\_012 after optimization program with using orthogonal aberrations

L_012 Opt.	Aberration functional			Balancing of aberrations	
	F	F1	Fh	F1%	Fh%
	19.07	0.75	18.32	4%	96%

It should be noted that to describe the aberration properties of this lens, it is necessary to use orthogonal aberrations up to the 11th order inclusive (of course, classical aberrations too), which also happens very rarely. Usually, to describe at east 80% of optical systems, it is sufficient to use aberrations up to the 7th order inclusive.

Contributions of aberrations 1,3,5, etc. orders in aberration functional F are as follows:

1	0.0045
3	0.0349
5	0.7478
7	0.1184
9	0.0737
11	0.0149

Maximum degree of aberration polynomial in variables  $r$ ,  $\rho$ ,  $\varphi$  are 11, 7, 4.

Maximum contribution in the aberration functional gives two orthogonal aberrations 5<sup>th</sup> order:

- primary field astigmatism 5th order -  $A_{102}R_4^2(r) R_2^2(\rho) \cos 2\varphi$  – contribution in F 23%;
- trefoil 5th order -  $A_{003}R_3^3(r) R_3^3(\rho) \cos 3\varphi$  - contribution in F 37%.

## 6. CONCLUSION.

This article sets out the directions of the 2nd level of education of the specialization optical systems design that are being formed at our university. This specialization contains a large amount of knowledge in optics, mathematics, design, technology and other related sciences, accumulated by many generations of outstanding optics - developers of optical systems. And the results and success of our work, as well as the memory of us in student's hearts, depend on our ability to transfer this knowledge to the younger generation, to form in them a love for this direction of optical science.

I want to note that I was lucky in my life with teachers. One of my first teachers was Slyusarev Georgy Georgievich. He was an outstanding Russian optician who developed in Russia the theory of 3th order aberrations and adapted its application in manual methods of designing initial optical systems. All generations of our opticians have studied and are learning from his books [3]. He was my academic adviser in graduate school, after which I defended my Ph.D. thesis on the topic "Study of the properties and application of Zernike polynomials in computational optics." I use his approach to the practical application of the classical theory of third-order aberrations in the development of automated synthesis of optical systems and teach this to students.

In conclusion, I would like to make a proposal to create a working group to form joint curricula for teaching the specialization of optical design theory and practice. I will gladly take part in the work of this group, especially since nowadays it is quite easy to organize any meetings and conferences online.

Thank you for attention.

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