Theory of optical imaging in the wave approach

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ABSTRACT

The imaging theory fully based on the wave approach is proposed. In the paraxial region an optical system can be divided into two independent parts. The imaging by the first part consisting of focusing elements is similar to the geometrical imaging. The consideration of the wave propagation through a second part being a set of distorters is sufficient to analyze the diffraction phenomenon occuring in the optical system. The outline of the theory followed by a few practical examples are given.

1. INTRODUCTION

The development of the coherent light technique requires a change of the imaging analysis method. The ray tracing and intensity distributions as the notions of geometrical optics ought to be changed into the wave and complex amplitude distributions. So far an exact imaging theory using the wave notions only has not appeared. However, apodized systems, in particular the propagation of Gaussian beam, require the wave analysis. In the student's consciousness two almost independent imaging models of optical systems have begun to operate. The imaging theory lost its consistence with the enlargement of the optical knowledge. In practice such a situation does not make a significant obstacle, because in many cases the geometrical approach brings sufficiently good conclusions, moreover almost all typical problems of the imaging in coherent illumination have been solved[1] with the aid of 4-f system shown in Fig.1. In this case an object complex amplitude distribution defined at the object focus plane π of an objective Ob1 generates its spectrum at the image focus plane π_F. An image of the object arises at the image focus plane π' of an objective Ob2 after a respective filtering of the spectrum. But such a rigid construction makes many doubts concerning the adjustment precision of all elements, the aberration influence of the objectives and the influence of their finite dimensions. Moreover a wave impinging onto an object placed at the plane π need not be planar, this means that the image of the point source S given by a condenser C need not be at infinity.

It is known fact, that the imaging by an infinitely large and aberration free objective Ob (fig.2) for a finite position of the image plane introduces a phase distortion.[2,3] A complex amplitude distribution from an object plane π reproduces

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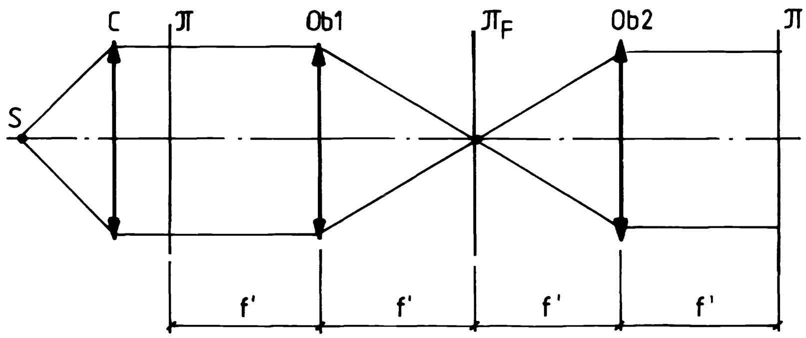


Fig. 1. 4-f imaging system

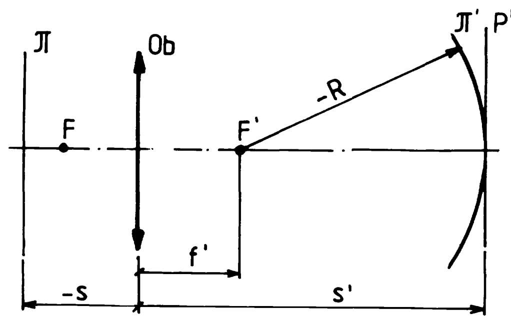


Fig. 2. A phase distortion of the image surface

with a proper magnification not at the image plane P' , but on the sphere π' with its centre at the image focus F' . The radius of the sphere is $R = \beta f'$, where β - a lateral magnification, f' - a focal length of the objective. Kogelnik proved[2] more general theorem for an infinity large and aberration free thin lens in the air. An object field distribution defined on a sphere with the centre at an axial point is reproduced by this lens in a different scale on an image sphere, the centre of curvature of the image sphere is at the image of the centre of curvature of the object sphere. The Fourier transform of an object generated by an objective in parallel rays (see Fig. 1) is the particular case of a more general Fourier transformation in a converging illumination.[4] These facts make elements of a more general imaging theory, which allows better comprehension of the role of the optical system in the imaging process. Moreover, the analysis of the influence of aperture stops and the aberrations of optical elements can also be included.

2. BASIC THEOREMS

In order to introduce the imaging theory in the wave approach some definitions and theorems have to be presented. They will be mentioned successively with comments to explain conditions related to them. Full proofs in references can be found.

A. A complex amplitude distribution $V(\bar{a})$ defined on a sphere Σ (Fig.3) with a centre at O generates its Fourier transform $V_F(\bar{R})$ on a sphere[5] Σ_F coincident with O. The centre O_F of the sphere Σ_F is coincident with the sphere Σ. This means that according to Fig. 3 we have

$$V_F(\bar{R}) = \frac{1}{\lambda r_0} FT^{-\bar{R}\bar{a}}\left[V(\bar{a})\right] , \quad (1)$$

where

$$\bar{R} = \frac{k\bar{\rho}}{r_0} . \quad (2)$$

$k = 2\pi/\lambda$, λ - wavelength, $\bar{a}$ and $\bar{\rho}$ are the radial vector coordinates for the spheres Σ and Σ_F , respectively. By FT^{-} the Fourier transform operator of two-dimensional function $V(\bar{a})$ is denoted.[6] The kernel of the transform is $\exp(-i\bar{R}\bar{a})$. The form of the parameterized coordinate $\bar{R}$ allows the use of the simple relation (1) for different distances r_0 .

Remarks: Eq.(1) is valid in the paraxial region and for $r_0 \gg \lambda$.

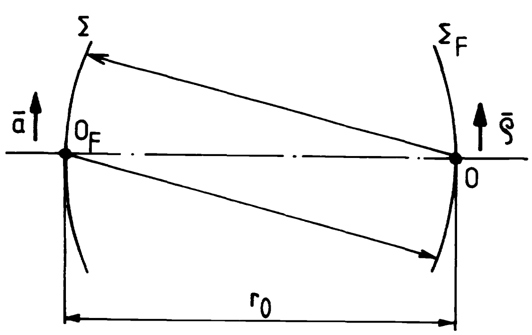


Fig. 3. The Fourier transform on a sphere Σ_F of an object defined on a sphere Σ .

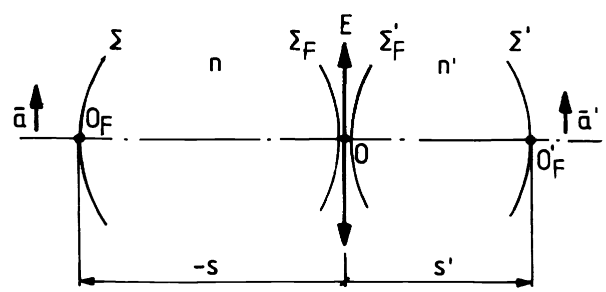


Fig. 4. The imaging between spheres Σ and Σ' by an elementary system E.

Imaging by a focusing system

B. Definition: A focusing system is an optical system with the transmittance equal to $\exp(ib\rho^2)$ for each value of radial coordinate ρ, where b is a constant.
Remarks: This is the theoretical assumption allowing to find an analogue in the geometrical imaging.

C. Imaging by an elementary focusing system E of a complex amplitude distribution $V(\bar{a})$ defined on a sphere Σ with the axial centre at O coincident with the system (Fig.4)

On the sphere Σ_F with the centre at O_F the Fourier transform of $V(\bar{a})$ arises, which repeats on the sphere Σ'_F with the centre at O'_F, where according to Fig.4 $n'/s' - n/s = n'/f'$. n' and n are refractive indexes of the image and object spaces, respectively. Because the complex amplitude distribution $V'(\bar{a}')$ on the sphere Σ' with the centre at O equals the Fourier transform of the field distribution from the sphere Σ'_F, using the known relation[6] $FT^{-}[FT^{-}[V(\bar{a})]] = 4\pi\, V(-a)$ and taking into account the scale problem, we can write

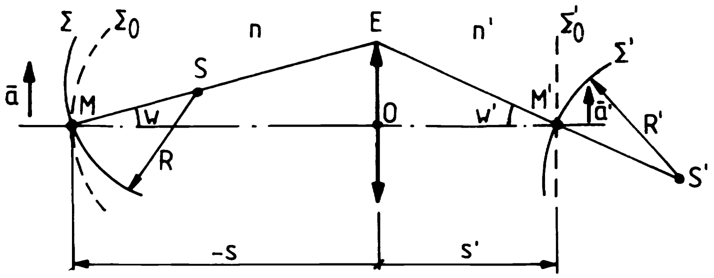
$$V'(\bar{a}') = \frac{1}{|\beta|}\, V\left(\frac{\bar{a}'}{\beta}\right) , \tag{3}$$

where the lateral magnification

$$\beta = \frac{ns'}{n's} . \tag{4}$$

Remarks: According to Eq.(3) the field distributions on the spheres Σ' and Σ differ only in their scales and the values of amplitudes (the coefficient $1/|\beta|$). It means that elementary focusing elements give the perfect imaging, in general, between spheres, not between planes. The theorem is more general than Kogelnik's one (see Introduction) , because it concerns a thin system with different object and image media. Such a generalization will allow in p.E the enlargement of the theorem under the consideration onto combined systems composed of an arbitrary number of elementary systems.

D. Using Eq.(3) for spheres Σ_O and Σ'_O with the common centre O (Fig.5) we can prove[7] that the same relation is fulfilled between field distributions on spheres Σ and Σ' with the centres at S and S' , respectively. The spheres Σ and Σ_O as well as the spheres Σ' and Σ'_O are coincident in pairs at the points M and M' , the points S' and S are optically conjugate. Additionally to Eqs. (3) and (4) a



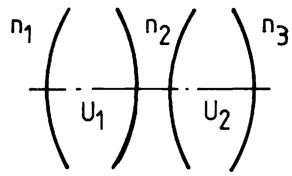


Fig. 5. The imaging between spheres Σ and Σ' by an elementary system E .

Fig. 6. The combination of two systems .

relation between the radii R' and R of the spheres Σ' and Σ can be found,[7] this means that

$$R' = \frac{n'}{n} \beta\beta_s R \ , \tag{5}$$

where β_s is the lateral magnification for the points S' and S .

Remarks: The relations (3) to (5) are valid for small angles w and w' (Fig.5).

E. If we assume that for systems U_1 and U_2 (Fig.6) the equations (3) and (5) are valid separately, then substituting these equations for the system U_1 into the same equations for the system U_2 , we can prove[7] that the equations (3) and (5) are valid for the combined system U_1+U_2 , too.

Ascertainment: The equations (3) and (5) are valid for any aberration free and infinitely large optical system composed of an arbitrary number of elementary systems. The equations concern the relation between field distributions defined on respective spheres, as it is shown in Fig.5 for an elementary system.

Examples: In Fig.7 some typical configurations of the imaging are given. If a point S (Fig.7a) represents a point source illuminating an object located in a plane coincident with a point M , then an object field distribution is defined on the sphere Σ_1 with the centre at S . The Fourier transform field distribution is given on the sphere Σ_2 with the centre at M . In the configuration shown in Fig.7a the Fourier distribution is virtual with regard to the object. The object and Fourier field distributions in the image space are given on the spheres Σ'_1 and Σ'_2 , respectively. Their centres are at the images of the points S and M denoted here by S' and M' . If the object is illuminated by a plane wave (source S at

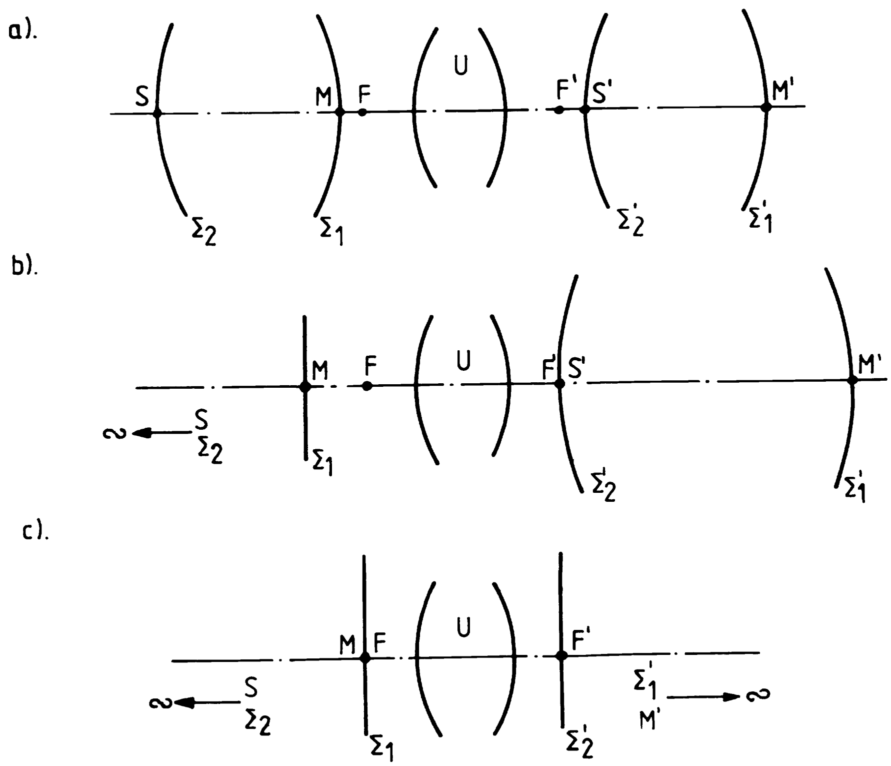


Fig. 7. Some typical examples of the imaging of object field distribution (between surfaces Σ_1 and Σ'_1) and its Fourier transform (between surfaces Σ_2 and Σ'_2).

infinity, Fig.7b), then the sphere Σ'_2 with Fourier distribution is coincident with the image focus of U (see Fig.2 for comparison). In Fig.7a the sphere Σ'_2 takes the form of a plane when the object is coincident with the object focus of U , because in this case the object M' is at infinity. If the point M is at the focus F and the source S at infinity simultaneously, then we meet the situation shown in Fig.7c, when the surfaces Σ_1 and Σ'_2 are planar. Such a configuration is a basic one to construct a 4-f system presented in Fig.1.

An object and its Fourier transform can be used interchangeably. In Fig.7 field distributions on the surfaces Σ_2 and Σ'_2 can be treated as the object distribution in the object and image spaces , respectively, and field distributions on the surfaces Σ_1 and Σ'_1 , as their Fourier transforms in the same spaces.

Imaging by a real system

A real optical system can be treated as a combined system of the focusing elements and the elements, the so-called distorters, for which transmittances are different from the transmittance of a focusing element.[8] For example in the case of an objective its focusing element represents the aberration-free part of the objective, and the distorter represents the aberrational part of it. Any diaphragms in optical system (aperture stop or lens mounts) are also distorters.

F. Distorters can be transferred by focusing elements to one of the spaces of the optical system under consideration.[8] The transfer condition is the equality of the transmittances of the transferring and transferred distorters at all conjugate points. If the distorters are transferred into the object or image space of the whole system, the wave propagation through a real optical system can be divided into two independent parts: propagation through a set of distorters and propagation through focusing elements. However, the imaging by a system of focusing elements is perfect, and the changes of field distributions introduced by such a system are given by a magnification of the system only. From the point of view of the diffraction analysis (influence of diaphragms and aberrations) the analysis of the wave propagation through a set of distorters only is sufficient to solve the problem of the wave propagation through a complex optical system.[8]

Remark: The conclusions presented are valid in isoplanatic regions of distorters.

Example: Typical imaging system in coherent illumination is shown in Fig.8a. An object P is illuminated through a condenser C by a point source G_o. An object field distribution for an objective Ob arises on a sphere Σ with the centre at G, where G is the image of G_o given by the condenser. To avoid a vignetting effect, as a rule, the image G has to be located at the plane of the entrance pupil Z of the objective. For the study of the imaging process by the objective we can neglect the source G_o, the condenser C and the object P assuming known field distribution on the sphere Σ only. However the objective can be treated as a combined system containing a focusing part U and a distorter D. The distorter represents the aberrational influence of the objective, the influence of the aperture stop and possible influence of the apodizing characteristic. To simplify our consideration at first the imaging by the focusing element U is taken. If points M

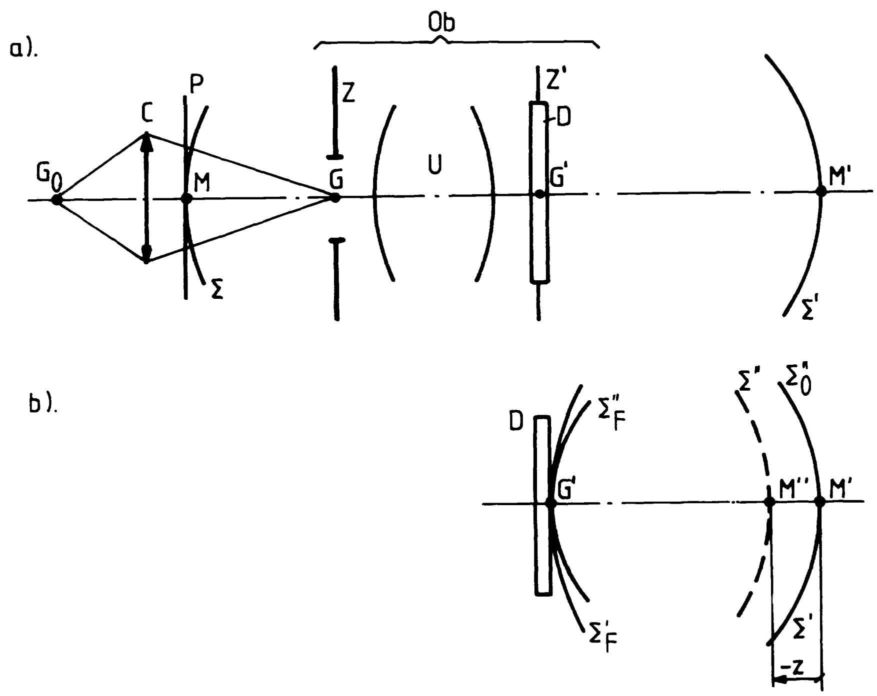


Fig. 8. The simplification of an imaging system (a) into the substitutional system (b).

and M' are mutually conjugate (Fig.8a) then the field distribution V from the sphere Σ is imaged on the sphere Σ' (the field distribution V') according to (3) in another scale only. Next the field distribution V' can be treated as the object distribution for the distorter D (see for comparison Fig.8a and Fig.8b). Because the imaging by the focusing element is perfect, then from the point of view of wave analyses it can be neglected and in this sense both imaging systems shown in Fig.8a i Fig.8b are equivalent. The evident simplification of the imaging system in Fig.8b is the result of the elimination of unessential factors of the imaging process.

To determine an image of the field distribution V' from the sphere Σ' given by the distorter D at first a field distribution V'_F on a sphere Σ'_F (with the centre at M') as the inverse Fourier transform of the field distribution V' must be found. Taking the Fourier transformation of the field distribution V'_F modified by the transmittance of the distorter D the image field distribution V" as the image given by D is determined. In general if the modification of V'_F concerns also a defocusing, then the image arises on a spheres Σ'' (Fig.8b), where a distance z is a defocusing measure. In the case of the focused imaging ($z=0$), the spheres Σ'' and Σ' in the figure are covered. The argumentation mentioned above leads[8] directly to the known convolution relation $V'' \propto V'_\delta \otimes V'$, where V'_δ is a field distribution in the point image given by the distorter.

Remarks: An analog simplification of the imaging consideration can be used for more complex optical systems to analyze a wave front distortion introduced by the wave propagation through aberrated elements.[9,10]

3. CONCLUSIONS

1. Using the wave approach only we can formulate some general principles concerning the imaging process in the paraxial region.

2. A focusing element or even a complex system combined of focusing elements changes a scale of a field distribution only. It concerns so called infinitely large and aberration free optical systems. The imaging laws are analogical to the geometrical ones, however, in general, the imaging occurs between spheres, not between planes.

3. In a real optical system its focusing elements and distorters can be separated and all distorters transferred to one space. The imaging by a set of focusing elements brings a change of a distribution scale only, and, as the imaging analogical to the geometrical one, it can be omitted in any diffraction analyses. Besides the scale change and the determination of the image sphere position, the analysis of the wave propagation through an isolated set of distorters is sufficient to find all diffraction properties of a complex optical system.

4. The synthesis of the optical system proposed simplifies considerably the considerations, moreover, it allows for students the better comprehension of the role of different parts of the optical system.

5. The analysis method proposed links in a natural way the more general wave approach with the geometrical imaging. After all, for incoherent illumination, there are not any differences between intensity distributions on spheres and planes, the imaging by a set of focusing elements is equivalent to Gaussian optics and geometrical rays can be found by the differentiaton of phase distributions. In particular the narrowness of the geometrical approach is emphasized.

6. The wave approach in the paraxial region is the first approximation allowing the analytical consideration, what is indispensable to lecture in coherent manner the optical knowledge about the imaging process. In practice the design of an optical system on a base of this consideration requires yet a numerical verification for real angles.

The lecture on the theory of optical imaging according to the approach presented has been given for students specializing in the optical engineering in Precision Mechanics Department of Warsaw University of Technology for several years. Whole knowledge and other ideas related to it are presented in the textbook.[11]

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