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Thermodynamic origin of nonimaging optics

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Abstract. Nonimaging optics is the theory of thermodynamically efficient optics and as such depends more on thermodynamics than on optics. Hence, in this paper, a condition for the "best" design is proposed based on purely thermodynamic arguments, which we believe has profound consequences for the designs of thermal and even photovoltaic systems. This way of looking at the problem of efficient concentration depends on probabilities, the ingredients of entropy and information theory, while "optics" in the conventional sense recedes into the background. Much of the paper is pedagogical and retrospective. Some of the development of flowline designs will be introduced at the end and the connection between the thermodynamics and flowline design will be graphically presented. We will conclude with some speculative directions of where the ideas might lead. [DOI: 10.1117/1.JPE.6.047003]

Keywords: nonimaging optics; thermodynamics; geometric flowline.

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1 Introduction

Nonimaging optics is the theory of thermodynamically efficient[1] optics and as such depends more on thermodynamics than on optics. It is by now a key feature of most solar concentrator designs. What is the best efficiency possible? When we pose this question, we are stepping outside the bounds of a particular subject. Questions of this kind are more properly in the province of thermodynamics which imposes limits on the possible (like energy conservation) and the impossible (like transferring heat from a cold body to a warm body without doing work). And that is why the fusion of the science of light (optics) with the science of heat (thermodynamics), is where much of the excitement is today. When the problem of maximal concentration from extended sources was first confronted,[2] the tools of Hamiltonian mechanics were utilized, because classical geometrical optics was concerned with "point sources."[3,4] In this paper, we first present the failure of classical point source optics. The purpose to repeat the illustration of this paradox is to show that the conventional point and line understanding of geometric optics cannot fully represent the nature of the physics behind modern optical designs. As the field (nonimaging optics) developed, it gradually became clear that the second law of thermodynamics was "the guiding hand" behind the various designs. If we were asked to predict what currently accepted principle would be valid 1000 years from now,[5] the second law would be a good bet. The purpose of this communication is to show how nonimaging optics can be derived from this principle. As a result, optics recedes into the background and we are left with abstract probabilities, the ingredients of entropy and information theory. This paper is organized as follows: Secs. 2–4 provide a brief review of nonimaging optics with the emphasis on its connection to thermodynamics. Sections 5–7 conclude with some speculative directions of where the ideas might lead, particularly how flowline can illustrate the thermodynamic origin of nonimaging concentrators.

2 Failure of the Imaging Optics

Conventional optics uses imaging ideas, or point sources, to represent the geometry of optical sources. This leads to conclusions in conflict with fundamental physics[6] (Fig. 1). In this paradox,

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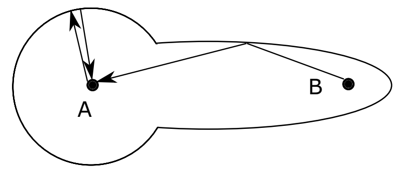

Fig. 1 The ellipse paradox: the ellipse images "point" object *B* (right) at point object *A* (left) "perfectly" and the sphere images *A* on itself perfectly.

the point object *A* is at the center of a spherical reflecting cavity, and it is also one focus of an elliptical reflecting cavity. The point object *B* is at the other focus. If we start *A* and *B* at the same temperature, the probability of radiation from *B* reaching *A* is clearly higher than *A* reaching *B*, as shown by the arrows. So we conclude that *A* warms up while *B* cools off, in violation of the second law of thermodynamics (heat goes only from higher temperature to lower temperature). The paradox is resolved by making *A* and *B* extended objects, no matter how small. In fact, a physical object with temperature has many degrees of freedom and cannot be point like. Then the correct cavity is not elliptical, but a nonimaging shape that ensures efficient equal radiation transfer between *A* and *B*.[6] It is worth mentioning that the correct nonimaging design does not converge to the ellipse/sphere configuration in the limit that the size of *A* and *B* tends to zero.

3 Nonimaging Optics, Designing Optimal Optics According to Thermodynamics

We can take a general concentration problem, as shown in Fig. 2, and ask the question of, what can be done to achieve the "best concentration"? In other words, what optics should be put into the box to achieve the maximum ratio between the areas of the aperture and the absorber?

$$C = \frac{A_2}{A_3}, \tag{1}$$

where C is the geometric concentration ratio and A is an area. In order to answer such a question, we have to make a reasonable assumption; all the energy from the radiation source that enters the aperture should reach the absorber.

$$Q_{12} = Q_{13}, \tag{2}$$

where Q represents the radiative heat (watts) that goes from one surface to another. A concentrator that does not meet such a requirement will have not achieved what is possibly the best. In other words, if two concentrators can both be designed to achieve the maximum radiation flux at the absorber, we would naturally choose the "better" concentrator which passes all energy from the aperture to the absorber instead of the one that is not capable of doing the same.

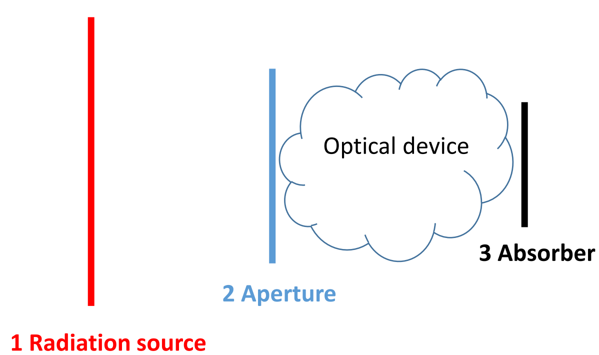

Fig. 2 Illustration of a concentrator, the optics between the aperture and the receiver is arbitrary.

No other assumptions will be needed. We are considering only the geometric optics, i.e., the radiative heat transfer is determined by the geometric setup and the shape of the optics, independent of the wavelength of the photons. (Dispersion would have to be considered differently or approximated with the dominant wavelength.) We can choose the objects to be of any temperature, and the result of the heat transfer due to the geometric optics should always satisfy the thermodynamic laws. Here, we pick a special case, i.e., the source and sink being both blackbody and at equal temperature. The aperture being a fully transmitting object can also be treated as a blackbody with the same temperature. The answer to the "best concentration" question can be found with the following thermodynamic arguments:

Second law demands that

$$Q_{12} = Q_{21}, \quad A_1 \sigma T^4 P_{12} = A_2 \sigma T^4 P_{21}, \quad A_1 P_{12} = \mathrm{A}_2 \mathrm{P}_{21}. \tag{3}$$

Here, P_{AB} is defined as the probability of heat from surface A reaching surface B, through any optical surface such as reflection, refraction, and so on. Or

$$P_{AB} = \frac{\text{number of rays reaching surface } B \text{ via optics}}{\text{number of rays emitted by surface } A}. \tag{4}$$

It is a more general concept compared with the idea of view factor in radiative heat transfer,[7] where only rays going from one surface directly to the other are considered.

Equation (3) represents the reciprocity of the radiative heat transfer or the second law of thermodynamics, which states that a cold object cannot heat up a hot object.

Similar to Eq. (3), we can conclude

$$Q_{13} = Q_{31}, \quad A_1 P_{13} = A_3 P_{31}. \tag{5}$$

From Eq. (2), or the first law of thermodynamics which states that energy is conserved, we can derive that

$$Q_{12} = Q_{13}, \quad A_1 P_{12} = A_1 P_{13}. \tag{6}$$

Combining Eqs. (3), (5), and (6), we conclude with

$$A_2 P_{21} = A_1 P_{12} = A_1 P_{13} = A_3 P_{31}, \quad A_2 P_{21} = A_3 P_{31}, \quad C = \frac{A_2}{A_3} = \frac{P_{31}}{P_{21}}. \tag{7}$$

For a lot of problems, P_{21} is predetermined due to the setup of the problem, e.g., solar concentration problems in which the sun subtends a certain angle. However, P_{31} can be manipulated with proper optical design. From Eq. (7), we find that the $C_{\max}$ is limited by $1/P_{21}$ and $C_{\max}$ can be reached when $P_{31} = 1$:

$$C \le C_{\max} = \frac{1}{P_{21}}. \tag{8}$$

The physical meaning of this is that an ideal concentrator limits all the "light" coming from the absorber to be within the range of the source. In other words, an ideal concentrator is also a perfect illuminator in which the illumination pattern has a sharp cut-off edge.

4 Tools to Design Thermodynamically Efficient Concentrator/Illuminators

Hoyt Hottel, an MIT engineer working on the theory of furnaces,[7,8] showed a convenient method for calculating radiation transfer between walls in a furnace using "strings." We now recognize this was much more than a shortcut to a tedious calculation, but instead the basis of an elegant algorithm for thermodynamically efficient optical design. In order to calculate the P_{21} from Sec. 3, we use the Hottel's strings on the radiation source 1 and aperture 2, as shown in Fig. 3

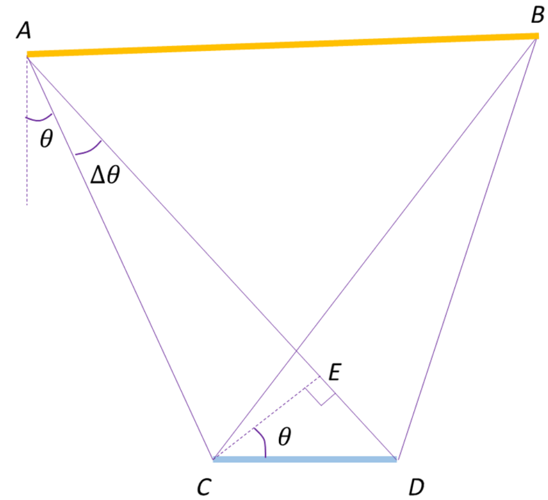

Fig. 3 The Hottel's strings of the source and aperture pair.

$$P_{21} = \frac{(\overline{AD} + \overline{BC}) - (\overline{AC} + \overline{BD})}{2\overline{CD}}. \tag{9}$$

As the source 1 recedes infinitely far away, $\Delta\theta$ approaches 0 and $\overline{AC} = \overline{AE}$. If we keep the setup symmetric, i.e., $\overline{AD} = \overline{BC}$ and $\overline{AC} = \overline{BD}$, then

$$P_{21} = \frac{2(\overline{AD} - \overline{AC})}{2\overline{CD}} = \frac{\overline{DE}}{\overline{CD}} = \sin(\theta), \quad C_{\max} = \frac{1}{P_{21}} = \frac{1}{\sin(\theta)}. \tag{10}$$

Equation (9) is exactly the same as the $C_{\max}$ derived with the etendue conservation,[2,9] implying a relationship between nonmiaging optics and thermodynamically optimal designs.

5 String and Flowline

Flowline is a vector field that can be defined in three-dimension (3-D) as[9]

$$\vec{J} = \left(\int \mathrm{d}p_y\,\mathrm{d}p_z, \int \mathrm{d}p_x\,\mathrm{d}p_z, \int \mathrm{d}p_y\,\mathrm{d}p_x\right), \tag{11}$$

where $\vec{J}$ is the flowline vector.[10,11] With a simple treatment of infinitely extrusion of a two-dimensional (2-D) cross section, one can find that the 2-D flowline vector is always bisecting the two extreme rays of the flowline source[9,12] (Fig. 4). In Fig. 5, a radiation source/sink pair (red line and green line) are shown. Flowlines can be traced from one radiation body to the other. For example, the blue solid lines are the flowline that starts at the edge of the green absorber (radiator), and they can be traced back to their source, the red radiator (absorber). If we trace back these flowlines from the edge of the other object to itself, the corresponding length h, which represents the etendue volume of the radiation heat transfer, are the same on either side. This also echoes the Kirchhoff's law that the second law of thermodynamics forbids the geometry of radiative heat transfer from being asymmetric.

Another look at the problem shows us that, because of the well-known property of a hyperbola, the difference of the distances to the foci remains constant (Hottel's string). The etendue between the radiation source and sink is also represented by the differences to the foci by Hottel's string Eq. (9). The reader might wonder how the ancient Greek mathematicians would feel about this connection between geometry and thermodynamics. To our knowledge, flowline is the closest realization of 3-D Hottel's strings. At least some of the 2-D flowlines, generalize to ideal 3-D systems.

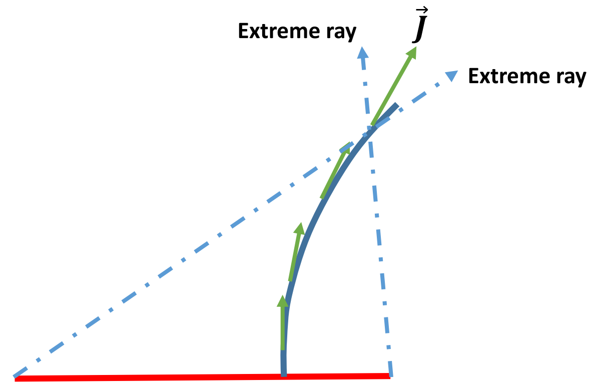

Fig. 4 Flowline of a line source in 2-D is hyperbola due to its property of always bisecting the two foci directions.

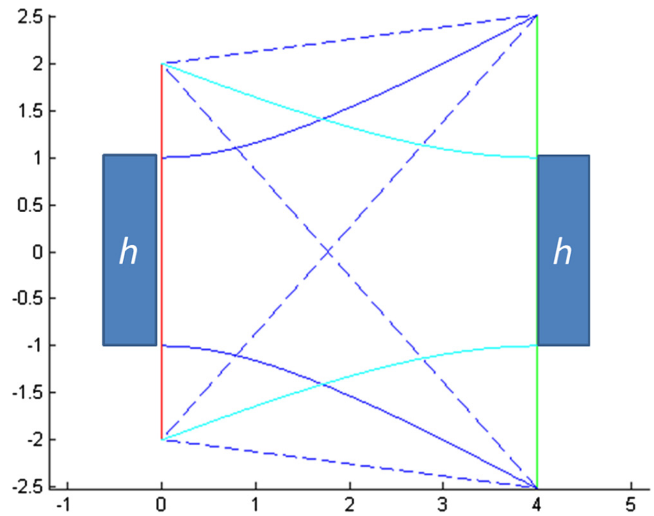

Fig. 5 Tracing the flowline between two lines.

6 Asymmetric Nonimaging Design

Although Eq. (9) is limited to the symmetric case of nonimaging design, many authors[13–16] have pointed out that the concept of nonimaging designs, or the thermodynamically optimal design that satisfies that $C_{\max} = \frac{1}{P_{21}}$, is not limited to symmetric cases (Fig. 6).

Here, 1 and 3 are predetermined radiation source and sink. To form an ideal concentrator with $C = C_{\max}$, a string $ac'c$ is tightly pinned down on points a, c, and point c' is moved, following an elliptical path to b'. Such a string method is consistent with the previous examples of compound parabolic concentrator (CPC).

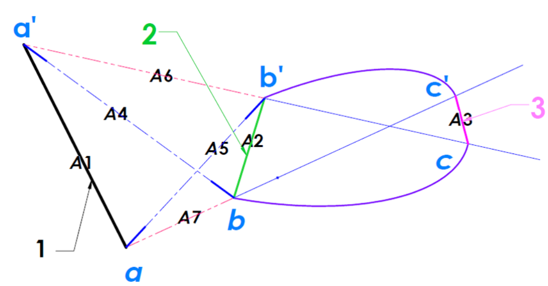

Fig. 6 The asymmetric application of the string method in CEC.

7 Linkage Between Flowlines and Mirrors

7.1 *Usage of Flowline as Mirror in Ideal Concentrators*

From Eq. (11), the flowline vector can be represented in a more suggestive form as $\vec{J} = \int \hat{n}\, \mathrm{d}\Omega$, which is the average direction of the energy flow.[17] This agrees with the flowline bisecting the rays from 2-D source, which also agrees with the well-known Snell's law of reflection (Fig. 7). In a 2-D setup, all mirrors will bisect flowline field[18] as shown in Fig. 7. The incoming blue ray is the extreme ray of a flowline source, i.e., the flowline source is between the directions of the incoming blue ray and the mirror, and possibly beyond. When calculating the flowline field infinitely close to the point of reflection, the two extreme rays will be the outbound reflected extreme ray and the extension of the extreme ray. The resulting direction of flowline that bisects these extreme rays is parallel to the mirror.

This connection between mirrors and flowlines can be utilized to construct ideal concentrators. Refer to Fig. 8, the flowline inside an ideal concentrator can be traced out by evaluating the average direction of all the rays from the flowline source. By tracing the rays of an ideal nonimaging concentrator such as shown in Fig. 6, one can calculate the flowline field according to its

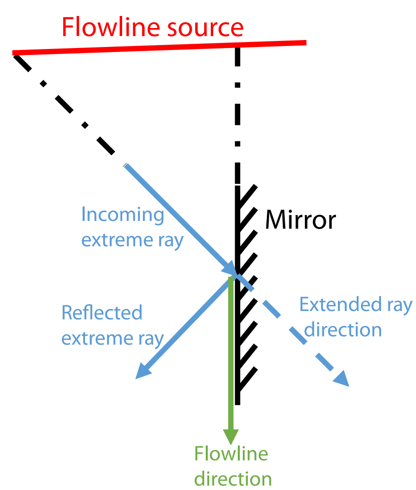

Fig. 7 The mirror also bisects the original ray's direction and its reflection direction same as the flowline direction.

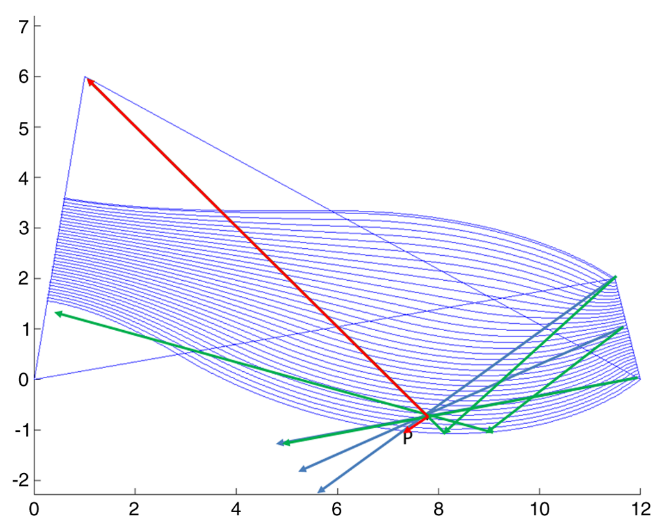

Fig. 8 The flowline inside a CEC.

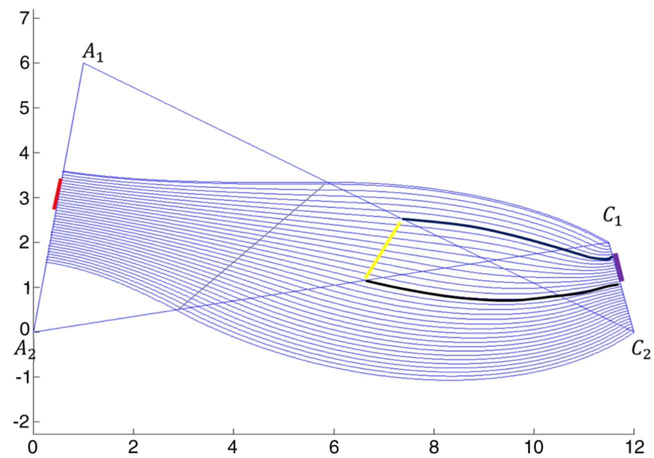

Fig. 9 Flowline ideal concentrator.

definition.[9] Any rays that can be traced back to the absorber, either directly from the absorber or by reflections, should be included in the flowline calculation. As an example here, an arbitrary point within the concentrator is chosen to calculate the flowline, as shown in Fig. 8. Rays directly from the source (blue arrows) or indirectly reflected by the mirrors (green arrows) can be both traced back to the absorber. The two extreme rays (red arrows) are noted and the direction bisecting them is the flowline direction. By tracing out these directions for each point within the concentrator, the flowline can be found to be controlled by the ideal concentrator [compound elliptical concentrator (CEC) in this case] to converge on the radiation source.

We can pick any pair of such flowlines and form an ideal concentrator. As shown in Fig. 9, the yellow line represents the aperture, the black lines represent the reflecting walls, and the purple line represents the absorber. The intriguing result is, we can trace the flowline and see how the ideal concentrator "guides" the radiation absorber onto a section in the radiation source (red line). Such a section has the same width of the radiation absorber, which implies that the etendue of the absorber is fully filled by rays coming from the source. This is equivalent to meeting the condition of $P_{31} = 1$, as required by the maximum concentration ratio Eq. (8).

7.2 *Thermodynamic Implication of the Flowline Mirrors*

If we cover the full length of the flowline with mirrors from the radiation absorber to the radiation source, then the etendue of the absorber is the same as that of the etendue between the mirrors at the source, and both are fully populated. In other words, the "etendue capacity" of both the purple area and the red area is fully occupied by the radiation coming from each other. Each of them sees only the other, not itself, not any other radiation source. This (as an etendue guide), however, will not concentrate, but it contains within it the elements to construct concentrators. By cutting the aperture at the points where flowlines are crossing over the diagonal lines (the end points of the yellow line), we get the concentrators (black lines). The reason for such a cutting position, is still unknown to us. This is a perspective of the ideal concentrators. This etendue transferring is interesting in itself.

One seemingly contradictory result of the flowline is the curious case of CPC flowline (Fig. 10). The flowlines right above the aperture of CPC are all parallel. If they continue to be parallel all the way to the radiation source, then the projected area by the flowline pairs on the radiation source will be the same as the aperture. This contradicts our previous prediction that it should be the same as the radiation absorber. The contradiction can be explained this way: the flowlines of CPC right above the aperture are, indeed, still hyperbolas. However, because the foci are far away from the radiation source, it appears to be "parallel," just the same as the hyperbolas with parallel asymptotes. As we follow the flowlines from the radiation source, in this particular limiting case, the hyperbolic flow lines become parallel lines at the vicinity of the aperture.

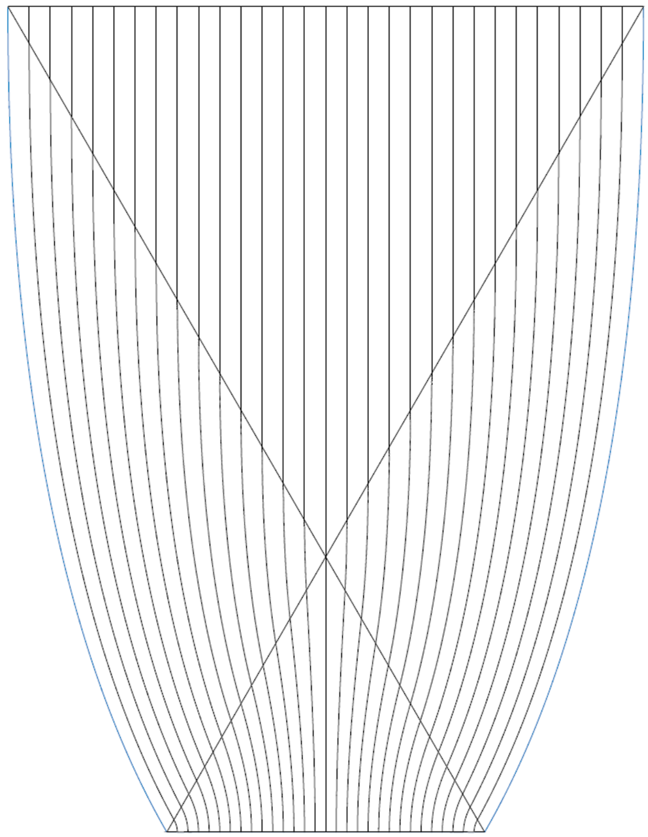

Fig. 10 Flowlines of CPC, being parallel and vertical at the aperture.

7.3 *Application of Thermodynamic Flowline*

In certain solar concentrator applications, not only the position of the sun is predetermined relative to the position of the absorber, e.g., due to the local latitude; the tilting of the aperture of the concentrator can also be limited to restrictions, such as shading or the covering glass. In the example shown in Fig. 11, the building integrated photo voltaic module (BiPV) may require the concentrator aperture to be also parallel to the wall, in order to minimize the shading between concentrators. By searching among the flowlines within the ideal concentrator BC and $B'C'$, as shown in Fig. 12, we can meet such a requirement by limiting the aperture to be parallel to the absorber. A simple binary search routine using starting points $C_0, C_1, \ldots$ for flowlines is shown in Fig. 12. In this process, the tilting of aperture $B'B_0, B'B_1, \ldots$ and so on,

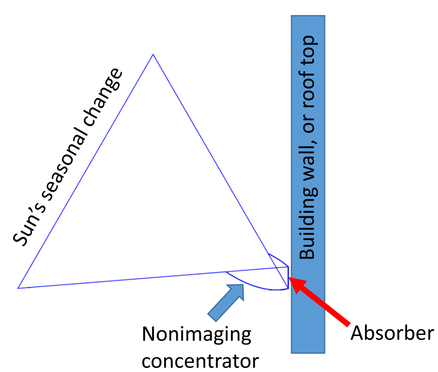

Fig. 11 The example of conventional nonimaging concentrator being unable to satisfy the restriction of BiPV modules.

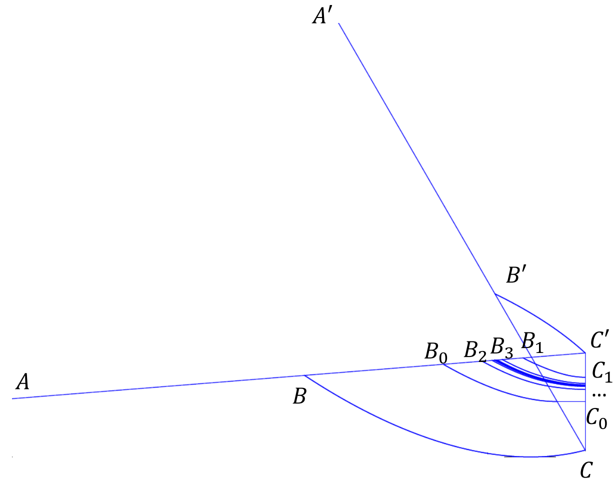

Fig. 12 By adjusting the starting position of the flowline within the absorber CC', we can adjust the angle of aperture BB'.

is compared with the angle of CC' and the program stops when the angle difference is within the tolerance of the design. This results in the concentrator shown in Fig. 13. By constructing an array of such concentrators, not only the relevant etendue at the aperture (the seasonal angle variation of the sun in this case, according to the full area of the wall) is fully used, but also the ideal concentration law of $C_{\max} = 1/P_{21}$ is also satisfied. The flowline in this case provided another degree of freedom to the ideal concentrator design by allowing the tilting angle of the

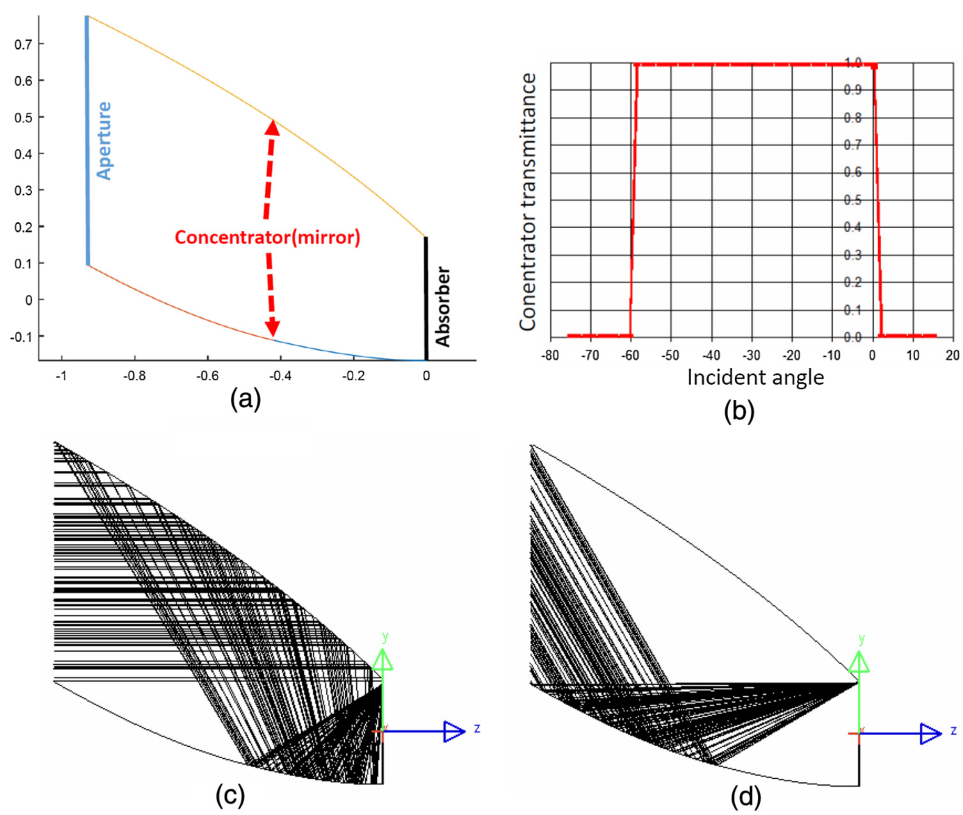

Fig. 13 The optical simulation of an ideal, nonimaging, asymmetric, flowline design, which meets the requirement of aperture being parallel to the absorber. (a) The concentrator constructed based on flowline, blue is a hyperbola curve, red and orange are elliptical curves. (b) The incident angle modifier shows that the transmittance response according to the angle is not symmetric, in this case, −60 deg to 0 deg.[20] (c) Edge ray tracing at 0 deg. (d) Edge ray tracing at −60 deg.

aperture also to be flexible. Such a result cannot be achieved by simply tilting the conventional CEC[19] or adding a secondary concentrator to the symmetric concentrator.[13] The detailed ray tracing can be found in Ref. 20.

8 Conclusion and Further Discussion

This paper has discussed the essence of ideal concentration. Thermodynamically speaking, the flux at the absorber surface cannot exceed the flux at the source surface. This is a fundamental principle that we cannot violate according to the second law of thermodynamics, even within the framework of geometric optics. Under the assumption that the most efficient concentrators will allow all the energy arriving at the aperture to be transmitted onto the absorber, we observe that the probability of any "virtual rays" coming from the absorber will also reach and only reach the absorber.

With the help of Hottel's strings and geometric flowlines, we demonstrated that at least some of the ideal concentrators have such a property: the flowline along the ideal concentrators will "guide" the etendue from the absorber to the source, and the region between the flowline, both at the source and at the absorber, is geometrically equivalent. This shows that flowline itself, being only under the constraints of geometry, is able to predict if a concentrator is ideal.

Furthermore, the flowline generated with the 2-D ideal concentrator can form infinitely more ideal concentrators. Specifically, any pair of such flowlines can construct an ideal concentrator which meets the requirement of $P_{31} = 1$ and $C_{\max} = 1/P_{21}$. Using this additional degree of freedom, we demonstrated how to form a flowline ideal concentrator according to an additional requirement: a certain tilting direction of the aperture.

We have seen that the Hottel's strings can be generalized with the geometric flowline. In some cases, this generalization prompts the question of its usage in 3-D, because unlike the Hottel's string, flowline is naturally 3-D. If one can successfully solve the problem of generating Hottle's string design using geometric flowline in the 2-D cases, one may be able to reapply the same principles into 3-D cases. In that sense, the flowlines open up the possibility of generalization of all current nonimaging optics 2-D design, which are constructed conventionally by Hottel's strings into three dimensions.

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Roland Winston is a leading figure in the field of nonimaging optics and its applications to solar energy. He is the inventor of the compound parabolic concentrator, used in solar energy, astronomy, and illumination. He is also a Guggenheim fellow, a Franklin Institute medalist past head of the University of Chicago, Department of Physics, a member of the founding faculty of University of California Merced, and currently, he is the head of the UC Solar.