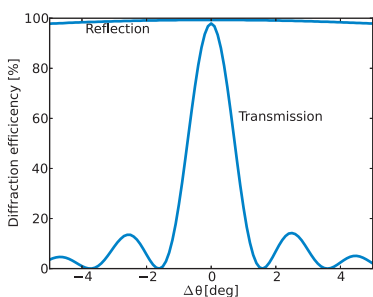
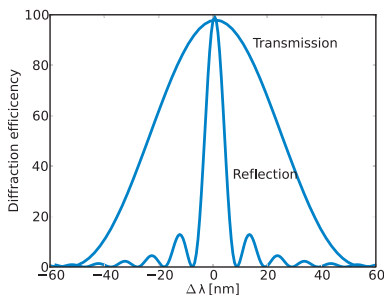


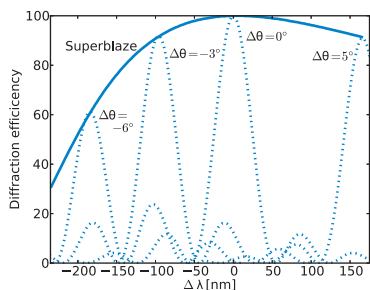
Dispersion of Thick-Volume Gratings

Even with the same material characteristics (thickness and modulation), the dispersion properties of **thick-volume gratings** are very different for reflection and transmission geometries. This is due to the significant difference in Bragg plane density ($1/\Lambda$) that is about 300–3000 lp/mm for transmission, and over 4000 lp/mm in reflection (for visible wavelengths). The variation of the diffraction efficiency according to the wavelength at the Bragg incidence angle is called the **blaze curve**.



As a general rule, thick-volume reflection gratings are wavelength-selective but angularly tolerant, which means that they diffract the same narrow-wavelength band (small $\Delta\lambda$) at any incidence angle (large $\Delta\theta$), making them useful as filters.

Thick-volume transmission gratings are wavelength-dispersive and angularly selective, which means that they diffract a large band of wavelengths (large $\Delta\lambda$) but in very specific directions (small $\Delta\theta$). When changing the incident angle, the efficiency for a particular wavelength drops and another one rises. The envelope of the entire efficiency spectrum according to the incidence angle is called the **super-blaze**. Transmission gratings are used as dispersive elements.



Remarkable Thin Gratings

Depending on the type and shape of the modulation, **thin gratings** have different diffraction properties. Transmittance is expressed in terms of the ratio of electric fields, and efficiency in term of the ratio of intensities.

Sinusoidal transmittance

- Modulation function: $t(x) = t_0 + \Delta t \sin(2\pi x/\Lambda)$
- Requirement: Must be positive, i.e., $t(x) \geq 0$
- Only three orders are diffracted: 0, +1, and -1. The maximum efficiency is obtained in the +1 and -1 orders when $t_0 = \Delta t = 0.5$ (peak-to-peak amplitude modulation = 1):

$$\eta_0 = t_0^2, \quad \eta_{\pm 1} = (\Delta t/2)^2 \ll 6.25\%, \quad \eta_{|m|>1} = 0$$

Square-wave transmittance

- Modulation function: $t(x) = t_0 + \Delta t \operatorname{sgn}(2\pi x/\Lambda)$
- Must be positive, i.e, $t(x) \geq 0$
- The diffraction efficiency in the first orders is higher than for a sinusoidal transmittance. The maximum efficiency is obtained when $t_0 = \Delta t = 0.5$ (peak-to-peak amplitude modulation = 1). There are no even orders:

$$\eta_0 = t_0^2, \quad \eta_{\pm 1} = (2\Delta t/\pi)^2 \leq 10.1\%, \quad \eta_{m=\text{even}} = 0, \quad \eta_{m=\text{odd}} = \frac{1}{m^2} \eta_{+1}$$

- Total diffracted energy: $\sum_{m \neq 0} \eta_m = \Delta t^2 \leq 24\%$

Sinusoidal phase

- Modulation function: $\varphi(x) = \varphi_0 + \Delta\varphi \sin(2\pi x/\Lambda)$
- The diffraction efficiency is expressed in terms of first-order Bessel functions:

$$\eta_0 = J_0^2(\Delta\varphi), \quad \eta_{\pm 1} = J_1^2(\Delta\varphi) \leq 33.8\%$$

- Maximum efficiency is achieved when $\Delta\varphi = 0.59\pi$ (peak-to-peak phase modulation = 1.18π)
- Total diffracted energy: $\sum_{m \neq 0} \eta_m = 1 - J_0^2(\Delta\varphi) \leq 100\%$

Remarkable Thin Gratings (cont.)

Square-function phase

- Modulation function: $\varphi(x) = \varphi_0 + \Delta\varphi \operatorname{sgn}(2\pi x/\Lambda)$
- The diffraction efficiency in the first orders is higher than for the sinusoidal phase. The maximum efficiency is obtained when $\Delta\varphi = \pi/2$ (peak-to-peak phase modulation = π). There are no even orders:

$$\eta_0 = \cos^2(\Delta\varphi), \quad \eta_{\pm 1} = \left[\frac{2}{\pi} \sin(\Delta\varphi) \right]^2 \leq 40.5\%,$$

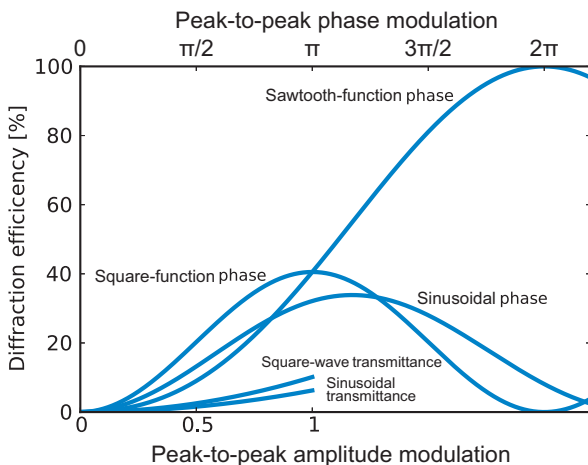
$$\eta_{m=\text{even}} = 0, \quad \eta_{m=\text{odd}} = \frac{1}{m^2} \eta_{+1}$$

- Total diffracted energy: $\sum_{m \neq 0} \eta_m = \sin^2(\Delta\varphi) \leq 100\%$

Sawtooth-function phase

- This corresponds to a blazed grating.
- Modulation function: $\varphi(x) = \varphi_0 + \Delta\varphi \sum_{m=1}^{\infty} \sin \frac{(2\pi m x/\Lambda)}{k}$
- The diffraction efficiency in the first orders can be up to 100% for $\Delta\varphi = \pi$ (peak-to-peak phase modulation = 2π):

$$\eta_{\pm 1} = \left[\frac{\sin(\Delta\varphi)}{\pi - \Delta\varphi} \right]^2 \leq 100\%$$



Scalar Theory of Diffraction: Kirchhoff Diffraction Integral

It would make sense to calculate the diffraction of light starting from Maxwell's equations of the electromagnetic field. However, the coupling between the electric and magnetic vectors rapidly complicates the equation, and analytic solutions can only be found for highly symmetrical cases.

The issue can be greatly simplified by replacing the electric-vector-wave equation E with a scalar equation $E(x, y, z, t)$:

$$\nabla^2 E = \left(\frac{1}{c^2}\right) \left(\frac{\partial^2 E}{\partial t^2}\right)$$

Because the magnetic component of the field is neglected, it is assumed that the diffraction does not affect the polarization of the incident wave.

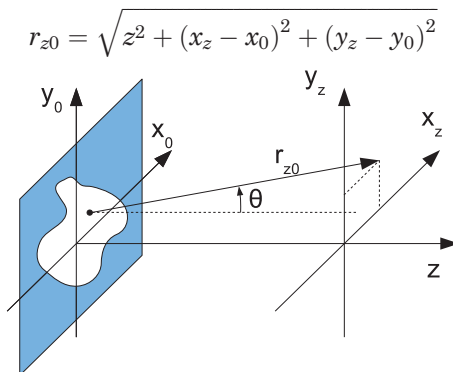
Two other assumptions are made as a basis for the **Kirchhoff diffraction integral**: The open portion of the aperture acts as a homogeneous source of the field $E_0(x_0, y_0)$, as stated by Huygens' principle, and the field is zero in the opaque portion of the aperture.

$$E(x_z, y_z) = \sum_{\text{aperture}} [\text{incident field at } x_0, y_0] \times [\text{wave propagation to } z : r_{z0}]$$

Note that the summation is over the aperture surface and is 2D. It translates into the Kirchhoff diffraction integral:

$$E(x_z, y_z) = \frac{1}{i\lambda} \int_{\text{aperture}} E_{(x_0, y_0)} \frac{\exp(ikr_{z0})}{r_{z0}} \cos \theta ds$$

where



Fresnel Diffraction Integral

From the Kirchhoff diffraction integral, some simplifications are possible. For example, consider the expansion of z in the Taylor series $\sqrt{1 + \varepsilon} = 1 + \frac{\varepsilon}{2} - \frac{\varepsilon^2}{8} + \dots$:

$$r_{z0} = z + \frac{1}{2} \left[\left(\frac{x_z - x_0}{z} \right)^2 + \left(\frac{y_z - y_0}{z} \right)^2 \right] + \dots$$

In a paraxial approximation, the aperture is small compared to the distance z : $z \gg x_z - x_0$ and $z \gg y_z - y_0$.

- The third term of the expansion is negligible even in the complex exponential. It must be much smaller than 2π .
- The second term of the expansion can be dropped in the denominator.
- $\cos \theta = 1$.

The simplified Kirchhoff diffraction integral is known as the **Fresnel diffraction integral**:

$$E(x_z, y_z) = \frac{\exp(ikz)}{i\lambda z} \int_{\text{aperture}} E(x_0, y_0) \exp \left[\frac{ik}{2z} (x_z - x_0)^2 + (y_z - y_0)^2 \right] ds$$

The near-field validity criteria of the Fresnel equation is

$$F = \frac{(D/2)^2}{z\lambda} \geq 1$$

where D is the aperture diameter, z is the distance of observation, λ is the wavelength, and F is the Fresnel number.

This equation is useful when z is large, considering the wavelength, but not necessarily much larger than the size of the aperture—thus the so-called “near-field” validity.